Digital Control of Magnetic Bearing with Rotationally Synchronized Interruption

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Summary

For the purpose of clean and high speed rotation, an actively controlled magnetic bearing has been developed. Recently, a digital control system has been used to get high performance by using the state variable feedback. However, the gyroscopic or inductive mutual effect of a rotating disc sometimes makes the high speed rotor unstable.

This paper introduces a new digital control scheme which utilizes the rotational interruption. The standard PID control is carried out with the constant time interval interruption, while the rotational interruption subroutine carries the static error cancellation and the cross coupling feedback to compensate for the undesirable mutual effect. This scheme is applied to a single rotor bearing system, and its capability is tested.

THE EXPERIMENTAL ROTOR AND ITS CONTROL SYSTEM

The Equations of Motion of the Rotor The equations of motion of the rotating shaft supported by the magnetic bearing are written in the following matrix form:

$$m \ddot{x} - \Omega J_p \dot{y} + k x = f_x - f_{bx} \qquad (1)$$

$$m \ddot{y} + \Omega J_p \dot{x} + k y = f_y - f_{by} \qquad (2)$$

where m and k are the mass and stiffness matrices for the x and y directions, and J_p is the gyroscopic moment matrix. The f_x and f_y are the external force vectors and f_{bx} and f_{by} are the supporting bearing force vectors /1-5/.

The Electro-Magnetic Force Positional feedback is used to maintain the neutral position of the shaft, while derivative and integral operations are widely used to improve the dynamic and static properties.

However the rotor is usually made of electro-conductive material which produces the eddy-current causing the undesirable Fleming force which is called as the inductive force. This effect produces the brake force to the rotation and the undesirable cross coupling effect. Hence, the disc is recommended to be made of laminated plates or ferrite material. For experimental convenience, we use only the solid iron to make the rotor, so that the inductive force cannot be neglected. Let us approximate the inductive force to be proportional to the relative velocity multiplied by the magnetic field. Then the electro-magnetic force can be written by

$$f_{bx} = K_p x + K_I \int x\, dt + K_D \dot{x} - K_1 \Omega y + K_2 \dot{x} \qquad (3)$$

$$f_{by} = K_p y + K_I \int y\, dt + K_D \dot{y} + K_1 \Omega x + K_2 \dot{y} \qquad (4)$$

where the first to third terms are the PID algorithm, while the fourth and fifth terms indicate the inductive force. The fifth term is the damping and can be neglected compared to the third term. However the fourth term indicates the cross coupling effect and affects adversely to the system stability /2,5/.

The Stability Analysis Equations (1) and (2) are applied to the experimental one degree-of-freedom rotor system. Hence, the displacement vector x and y can be simplified by the scalar. Applying the PD control of eqs. (3) and (4), we have

$$m \ddot{x} - \Omega K_J J_p \dot{y} + K_D \dot{x} + (k + K_p) x - K_1 \Omega y = 0 \qquad (5)$$

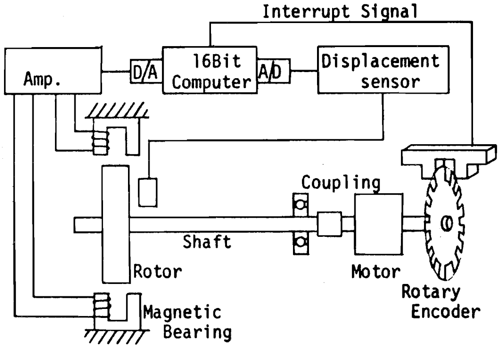


Fig. 1. The scheme of experimental setup

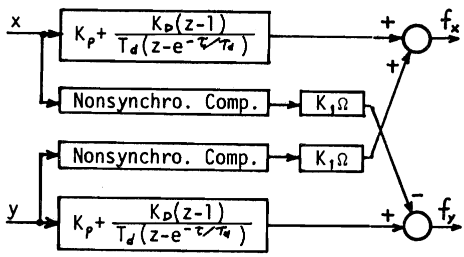


Fig. 2. The block diagram of the proposed digital control

$$m\ddot{y} + \Omega K_J J_p \dot{x} + K_D \dot{y} + (k + K_p) y + K_I \Omega x = 0 \qquad (6)$$

where K_J is the gyroscopic factor determined by the vibrating mode. The velocity feedback K_D improves the damping of the system. Hence the fundamental pole of $\sqrt{(k+K_p)/m}$ will be damped by increasing the gain K_D.

However, the cross coupling effects are apt to make the system unstable. The gyroscopic effect will increase the frequency of one of the duplicated poles and decrease the other, and are recognized as the forward and backward precessional motion. When the backward precessional frequency approaches zero, the system is apt to be unstable. A cross feedback technique has been introduced to correct this difficulty /5/. A more harmful mutual effect is the inductive force which will move the duplicated poles away one to the right and the other to the left-half plane. This means that increasing $K_I\Omega$ makes the system unstable. Cross feedback is highly desirable to cancel out the undesirable cross coupling effects.

DIGITAL CONTROL SYSTEM

The PID Control The displacement of the rotor can be measured without phisical contacts, and fed back to the magnetic bearing to maintain the neutral shaft position. The derivative operation improves the dynamic stability and integral feedback decreases the static error. Hence the following analog transfer function is widely used:

$$G_c(s) = \frac{K_I}{s + 1/T_I} + K_P + \frac{K_D s}{1 + T_D s} \qquad (7)$$

where T_I and T_D are the time constants of the derivative and integral circuits.

The equivalent discrete transfer function is given by

$$G_c(z) = \frac{K_I}{z - e^{-\tau/T_I}} + K_P + \frac{K_D(z-1)}{T_D(z - e^{-\tau/T_D})} \qquad (8)$$

where τ is the sampling interval. This digital PID control is carried out by the constant time interrupt subroutine which is shown in Fig. 3.

The Rotationally Synchronized Interruption As mentioned before, the cross coupling effects will affect adversely to the high speed stability of the rotating shaft. The most preferable technique is the use of cross feedback

$$G_m(s) = (K_J J_p s + K_1)\Omega \tag{9}$$

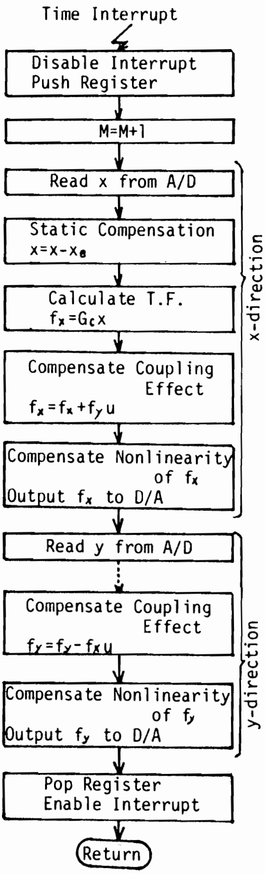


Fig. 3. The time interrupt subroutine

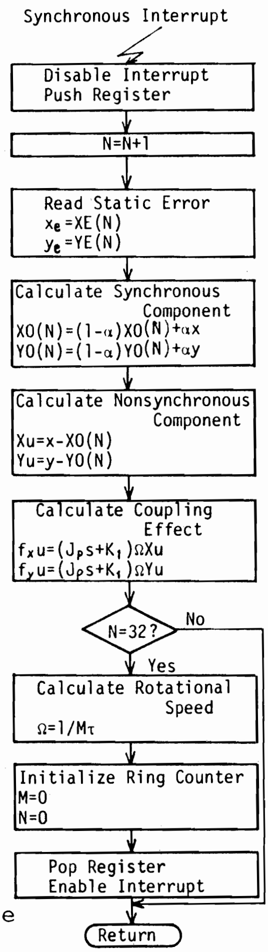


Fig. 4. The rotational interrupt subroutine

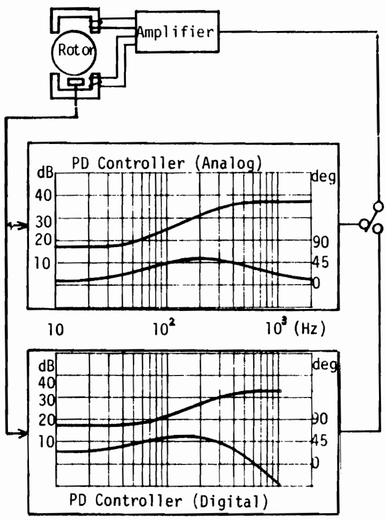


Fig. 5. The comparison between the analog and digital control

to cancel out the undesirable coupling effects. This means, however, that the feedback gain should change proportionally to the rotating speed Ω. The digital controller can easily measure the rotating speed by means of rotationally synchronized interruption: that is, the number of time interruptions M is counted for one revolution of the shaft. Then the rotating speed can be calculated by $\Omega = 1/M\tau$.

Another important technique is to separate the synchronous and asynchronous components from the measured signal. The synchronous component can be used to compensate for the static error of the measured signal. The eddy-current displacement sensor sometimes produces the synchronized error caused by the nonhomogeneity of the measured surface material. This error is premeasured and stored in the ring memory XE(N). For each interruption N (in this case N=1, 2, $\cdots$,32), the displacement x is subtracted by this error XE(N) to produce the true displacement.

The cross coupling effect increases when the shaft runs at very high speed. Hence, the unstable frequency is usually several times lower than the rotating speed. The cross feedback signal should preferably be made from the asynchronous low frequency component. In the interrupt subroutine, the synchronous component XO(N) can be made by exponentially averaging the measured signal.

$$XO(N) = (1 - \alpha)\, XO(N) + \alpha\, x \qquad (10)$$

where α ($0<\alpha<1$) is the averaging constant. Then the asynchronous component Xu can be made by subtracting the synchronous component XO(N) from the measured signal x.

$$Xu = x - XO(N) \qquad (11)$$

This signal is used to make the cross feedback signal. The rotational interrupt subroutine is shown in Fig. 4. The block diagram of proposed digital controller is shown in Fig. 2.

EXPERIMENTAL RESULTS AND CONSIDERATIONS

<u>The Experimental Apparatus</u> The scheme of the experimental magnetic bearing system is shown in Fig. 1. The shaft is made of

4 Φ × 200 mm stainless steel with an iron rotor of 78.5 g. This solid rotor is supported by a magnetic bearing which is converted from a stator of a 4 phase stepping motor. Hence, the inductive force cannot be neglected. The rotor has a 0.7 mm air gap and can run up to 1047 rad/s (10000 rpm) by a DC motor. A 32 step interrupt encorder is mounted on the other side of DC motor.

PD Control The critical speed of the rotating shaft was analyzed by means of the transfer matrix method and the results are shown in Fig. 6. The bearing stiffness is 2600 N/m, which causes the first critical of 180 rad/s (1720 rpm) and the second of 2530 rad/s (24100 rpm). This paper uses PD control of x and y directions separately. The frequency responses of the analog and digital controller(τ=1 ms) are shown in Fig. 5. The unbalance response is shown in Fig. 7 and the orbital trajectory where the shaft is running at 1043 rad/s with PD controller is shown in Fig. 8 (a). These experiments show that the shaft can be stabilized and can rotate up to 1043 rad/s.

Simular results are obtained by the digital PD controller.

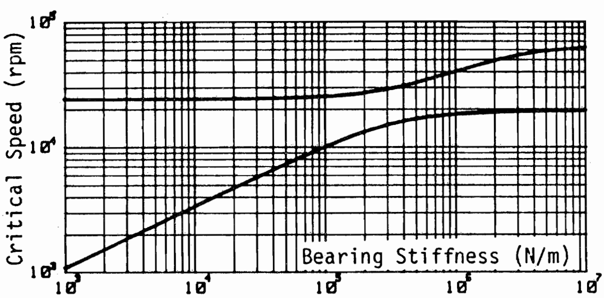
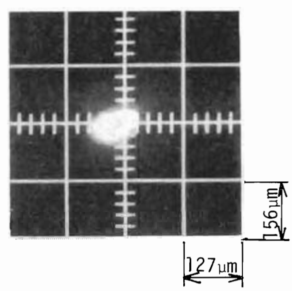
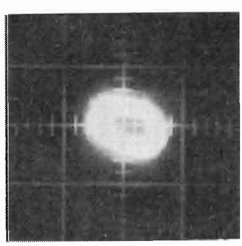


Fig. 6. The critical speed versus bearing stiffness



(a) Analog control



(b) Digital Control

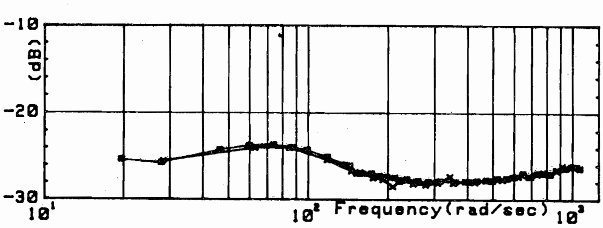


Fig. 7. The unbalance response with analog PD control

Fig. 8. The orbital trajectory with the PD control

The orbital trajectory at 1043 rad/s is shown in Fig. 8 (b), which is a little larger than the analog controlled case. The frequency response is shown in Fig. 9. In this case, the rotor is running at 280 rad/s. The fundamental rotating peak at 280 rad/s and the second one at 560 rad/s can be recognized. However, the response indicates high stability of this system.

The Rotational Interrupt Control In addition to the standard PD control, rotationally synchronized interruption and cross feed-back control is applied to the magnetic bearing. The system is apt to be unstabilized by the electro-inductive force. The example of orbital trajectory which indicates the subharmonic vibration is shown in Fig. 10. Decreasing the velocity feedback gain, a lower frequency unstable vibration occurs. Case (a) indicates a 100 rad/s unstable vibration with analog control, while (b) shows the 175 rad/s unstability with digital control.

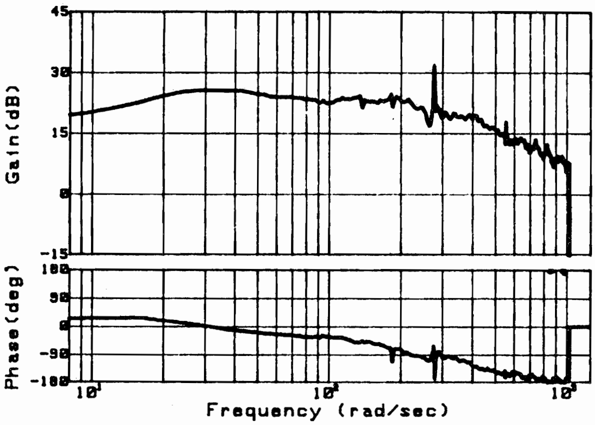
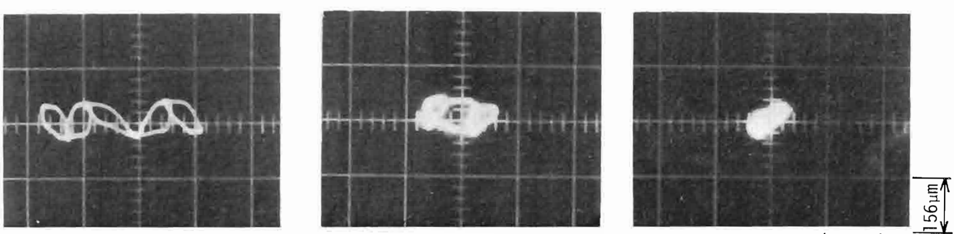


Fig. 9. The frequency response with digital PD control



(a)Analog control (b)Digital control (c)Rotationally interrupt control

Fig. 10. The comparison of the orbit trajectories

A rotational interrupt control is carried out with a standard 8086 microprocessor which accepts 32 interrupt pulses per revolution as shown in Fig. 1. For each interruption, a subroutine calculates the asynchronous component of the measured signal which is crossly fed back to the rotor to cancel out the undesirable inductive force. The stabilized orbital trajectory is shown in Fig. 10 (c).

CONCLUSIONS

A simple one degree-of-freedom magnetic bearing rotor system is controlled by a digital control algorithm installed in a 16 bit microprocessor. The following conclusions are obtained:

(1) A digital PD controller can improve the dynamic stability of the rotating shaft as well as the analog one. Soft support and high damping control is preferable.

(2) A new digital control algorithm is applied which can compensate for the undesirable cross-coupling effect of the radial directions. A rotationally synchronized interrupt and its subroutine can measure the rotating speed and asynchronous component which can be utilized to compute a cross feedback signal.

Further work is continuing to apply this cross feedback technique to the multi degree-of-freedom rotor system and to clarify its ability to compensate gyroscopic effects.

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