Optimal Design of Structure Predefined Discrete Control for Rotors in Magnetic Bearings (SPOC-D)

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ABSTRACT

Many layout techniques of time-discrete control algorithms for magnetic bearings are based on quasi-continuous approximations of traditional time-continuous controllers, although this approach does not provide the full dynamic range and may even lead to instability.

State-feedback methods require an observer since the number of measured signals is usually less than the number of states in the mathematical rotor model. In case of flexible rotor structures this observer approach leads to a high-order fully coupled controller and often shows untolerable parameter sensitivity.

The goal of this paper is to present a layout method for optimal discrete dynamic compensators with structural constraints typical in magnetic bearing applications, i.e. a predefined controller order or a decentralized feedback structure, in order to fill the gap between well known PD-algorithms and state-LQ-schemes. Similar to the latter method the optimal feedback coefficients are obtained by minimization of a quadratic performance index. Both the performance index and the corresponding vector gradient can be computed easily for every set of feedback parameters. Quick convergence can be achieved by a powerfull numerical optimization routine.

Results of a **SPOC-D** (**S**tructure-**P**redefined **O**ptimal **C**ontrol for **D**iscrete systems) layed out simple magnetic bearing system are presented and compared with the system properties obtained by standard controller design methods.

1. INTRODUCTION, GOAL OF THIS PAPER

Active electromagnetic bearings show a number of interesting characteristics and are more and more applied for new solutions to classical machine dynamic problems. Due to the rapid development in microprocessor technology, digital control makes it nowadays possible to take advantage of the wide range of achievable magnetic bearing parameters (i.e. damping and stiffness). Even changes in the structure of the controller can easily be made by varying the controller software instead of loosing much time altering the controller hardware.

The choice of the controller structure depends on several aspects of the magnetically borne rotor system and has to be carefully evaluated to achieve the desired performance properties. These aspects are:

- rotor and corresponding rotor model,
- number of controlled bending modes and overall dynamic properties,
- number of available output signals,
- performance of the microprocessor.

A fundamental property of magnetic bearing control is the fact that the number of measurement signals is usually less than the number of states in the corresponding model description. Therefore optimal pure state-feedback is not possible.

Basically, there are two ways to handle the problem of too few outputs and of getting satisfactory controller performance: one possibility is the use of a complete state-feedback with a full or reduced-order observer to estimate the missing velocities and, in case of higher system orders, all the residual states. This approach leads to a high-order fully coupled controller with the consequence of lower sample rates due to the given computational power of the controller hardware. In addition to this disadvantage untolerable parameter sensitivities might occur implementing high-order observers (/1/).

A second way to cope with the problem of too few output signals is the predefinition of the dynamic compensator structure, i.e. the predefined discrete controller consists of a given number of time lag elements with given interconnections. This output feedback approach leads, for example, to a decentralized control where every output path is directly connected to a specific input path without any further interconnections. Decentralized continuous controllers are a classical feedback structure. Optimization of these controller types has been investigated by LEVINE/ATHANS (/2/) and SENNING (/3/) and specifically for their application in magnetic bearing systems by BLEULER (/4/).

The practical application of decentralized time-discrete control schemes is an often followed design approach. The necessary control parameters are usually obtained by a quasi-continuous approximation of the corresponding continuous parameters. In case of magnetic bearings, however, it can be shown that much better sets of control parameters can be found if a parameter optimization is done directly in the time-discrete state space and not based on quasi-continuous approximations. Furthermore, even better optimization results can be achieved if the controller structure is slightly changed, so that a quasi-continuous approach to determine the feedback coefficients is not possible any more.

The **SPOC-D** method (**S**tructure-**P**redefined **O**ptimal **C**ontrol for **D**iscrete systems) presented in this paper provides a way of designing controllers by parameter optimization. Similar to state-LQ-methods the optimal feedback coefficients for the structurally constrained system are obtained by minimization of a quadratic performance index involving both the system states and the control inputs. The numerical minimum search can be done quite easily due to the fact that both the performance index and the corresponding vector gradient are given in an analytic form for every set of feedback parameters. An additional feature of the SPOC-D method is the possibility of optimization under several constraints as prescribed static bearing stiffness or consideration of symmetries. Furthermore, SPOC-D can minimize a multi-model performance index to ensure stability over a wider range of changing plant parameters.

A simple example of a magnetically borne elastic rotor system is presented (fig. 3.). A quasi-continuous controller design method is compared with the SPOC-D method for a layout of an optimal set of control parameters. Results are illustrated by simulations of the system behaviour.

2. OPTIMIZING STRUCTURE-PREDEFINED DISCRETE CONTROL

2.1. PLANT, CONTROLLER STRUCTURE AND PERFORMANCE INDEX

A general time-discrete state space description of a linear dynamic system is given by

$$x_{k+1} = A\,x_k + \sum_i \mathbf{b}_i\,u_{ki} \tag{1}$$

$$y_{ki} = C_i\,x_k \qquad (i = 1\ldots) \tag{2}$$

In case of structure predefined control the state vector of the plant is augmented by the states of the dynamic compensator to a global state vector x_k. The global system matrix A describes the dynamic behaviour of the plant and includes the predefined interconnections between plant and compensator. The scalar input signals u_i are called *control stations*. For each control station a single input column vector $\mathbf{b}_i$ describes its influence on the dynamic system. Furthermore a special *observation station* y_{ki} is assigned.to each control station

The structure predefined control loop will be closed by the unknown output feedback parameters d_{ij} included in the single row vectors $\mathbf{d}_i$.

$$u_{ki} = \mathbf{d}_i\,y_{ki} = \mathbf{d}_i\,C_i\,x_k \qquad (i = 1\ldots) \tag{3}$$

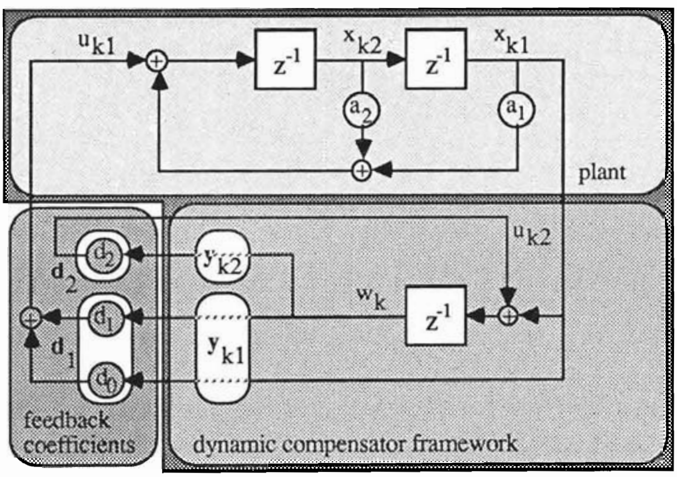

fig. 1. plant augmented by a structure-predefined dynamic compensator for direct output feedback

Fig 1. shows a simple example of a dynamic system as shown in (1, 2, 3): the plant description of order two (x_1, x_2) is augmented by a predefined dynamic compensator of order one (w). Two output signals (y_1, y_2) are available for the controller specified by the three unknown parameters (d_{10}, d_{11}, d_{22}). For simplicity, the station index *i* for the coefficients d_{ij}

will be omitted in all further examples (as already shown in fig. 1.). The special form of (1, 2) for the given example is shown in (1a, 2a).

$$\begin{bmatrix} x_1 \\ x_2 \\ w \end{bmatrix}_{k+1} = \begin{bmatrix} 0 & 1 & 0 \\ a_1 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ w \end{bmatrix}_k + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_{k1} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_{k2} \tag{1a}$$

$$y_{k1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} x_k \quad ; \quad y_{k2} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x_k \tag{2a}$$

The closed-loop structure for the given example is an outer feedback u_{k1} for the plant and an inner feedback u_{k2} for the dynamic compensator. Thus we get

$$u_{k1} = \begin{bmatrix} d_0 & d_1 \end{bmatrix} y_{k1} \quad ; \quad u_{k2} = \begin{bmatrix} d_2 \end{bmatrix} y_{k2} \tag{3a}$$

with the unknown controller coefficients d_0, d_1 and d_2 that have to be determined by optimization of a performance index in order to guarantee overall stability and satisfactory dynamic behaviour of the closed-loop system (1, 2, 3). For this purpose it is assumed that the dynamic system (1, 2) is completely observable and controllable.

For the subsequent optimization of the described output feedback it will be very important that the parameters d_{ij} of each feedback vector $\mathbf{d}_i$ must be independent of each other: each coefficient d_{ij} can appear only once at one given place in one feedback vector $\mathbf{d}_i$. Furthermore, the dynamic compensator structure should be formulated in a suitable normal form and should not be over-parametrized.

As described in (/2/, /3/) the performance index PI for the closed-loop system (1, 2, 3) is formulated as a quadratic form in the states $\mathbf{x}_k$ and in the control stations u_{ki} involving the weighting matrix $Q \geq 0$ and the scalar weighting factors $r_i > 0$:

$$PI = \sum_{k=0}^{\infty} (x_k^T Q x_k + \sum_i u_{ki}^T r_i u_{ki}) \quad \text{--->} \quad \min \tag{4}$$

Fig 2. now shows the four necessary fundamental equations ((5)-(8)) for optimality of the performance index (4). They differ slightly from the time-continuous case. The derivations are briefly explained below, but not shown in detail in this paper. The exact steps will be presented in (/5/).

At first the performance index is brought into an equivalent form with the use of the initial state $\mathbf{x}_0$ and the closed-loop matrix A_{clsd}. The infinite sum can then be replaced by a Lyapunov equation whose solution P determines PI together with the initial state $\mathbf{x}_0$. Applying small perturbations $\mathbf{dd}_i$ to the feedback vectors $\mathbf{d}_i$ a perturbed performance index PI + dPI is found which finally leads to an analytic form for the vector gradient of the performance index together with a dual Lyapunov equation.

$$A_{clsd} = A + \sum_i b_i d_i C_i \tag{5}$$

$$A_{clsd}^T P A_{clsd} - P + Q + \sum_i C_i^T d_i^T r_i d_i C_i = 0$$

$$PI = \text{trace}\,(PX_0) \tag{6}$$

$$X_0 = x_0^T x_0$$

$$A_{clsd} X A_{clsd}^T - X + X_0 = 0 \tag{7}$$

$$\frac{\partial PI}{\partial d_i} = 2\left(b_i^T P A_{clsd} + r_i d_i C_i \right) X C_i^T = 0 \quad (i = 1 \ldots) \tag{8}$$

fig. 2. necessary optimality equations to minimize the performance index PI

2.2. Minimization of the Performance Index Under Constraints

The prescription of the static bearing stiffness is typical for magnetic bearing applications, so that the output feedback coefficients d_{ij} *are no longer independent* of each other.

Any parameter interdependences of this or a different kind can be considered in SPOC-D by including additional control stations in (1), the feedback coefficients of which are linear or nonlinear functions of the residual set of independent parameters d_{ij}. The effect on the optimality equations ((5)-(8)) are additional terms for the closed-loop matrix A_{clsd}, the Lyapunov equation for P and for the vector gradient $\partial PI/\partial \mathbf{d}_i$.

Note that all constraints are considered in the modified necessary equations for optimality and not by a special algorithm for minimization under constraints.

Although these modified optimality equations are of much more interest to the field of technical applications, they are not shown in this paper, but will be well described in (/5/). All results for the example below, however, have been found with the modified equations for optimization under constraints.

3. Numerical Minimization Procedure

The necessary matrix equations for optimality of the performance index ((5)-(8)) have to be solved simultaneously. They are coupled and highly nonlinear. Since it is not possible to find a closed form for the minimum of the performance index PI the solution has to be found by means of a powerful numerical method including the ability to solve Lyapunov equations, which is found in many available software packages for matrix handling (e.g. CTRL-C). The fact of having the vector gradient in an analytic form is of great advantage for the minimization.

The numerical procedure used to find the results given below is a special *Quasi-Newton-Method* called *Davidon-Fletcher-Powell-Method* including self-scaling and restarting. It is described in (/6/). The method requires an additional one-dimensional minimum search algorithm which, in our case, is implemented as a cubic fit.

4. RESULTS

The figure below shows a simple magnetically borne rotor system. It is assumed that the elastic rotor can only move in one plane. Thus the total number of degrees of freedom is three if the mass of the elastic shaft is neglected. Gyroscopic effects are not considered. The system is geometrically symmetric, though the shape of the rotor movements may not be symmetric.

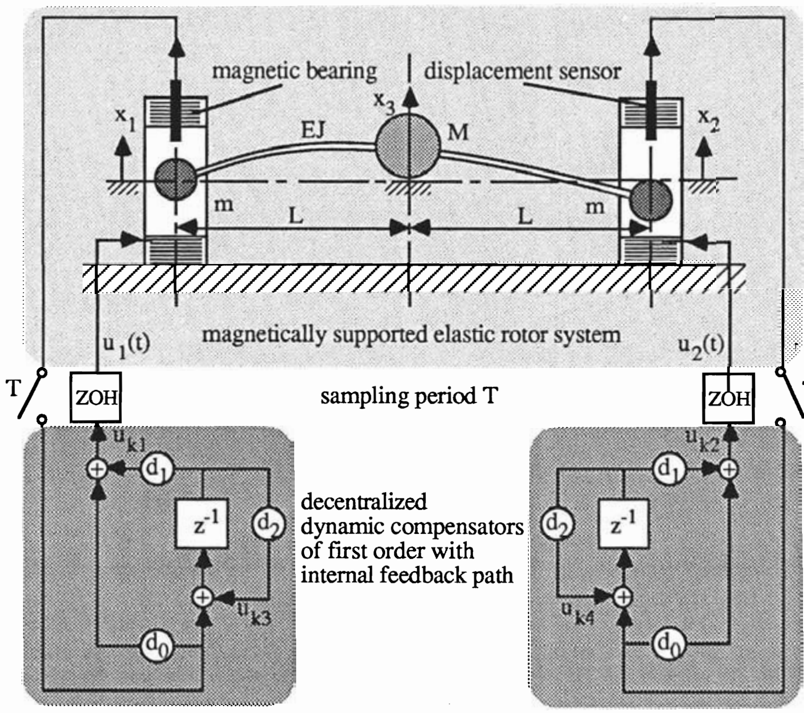

fig. 3. elastic magnetically borne rotor system augmented by two time-discrete decentralized dynamic compensators of predefined structure

The rotor system has two input signals: the force or electric current in each magnetic bearing. Only two sensors measure the rotor position in each bearing, which is most frequently the case in technical applications.

The state space dimension of the plant description is six; thus four system states are not measured. The control loop is closed augmenting the plant description by two decentralized dynamic compensators to stabilize the global system and to achieve a "good" dynamic performance. Each decentralized controller can be described by a general first-order transfer function involving the coefficients d_0, d_1 and d_2. This controller structure predefinition does not correspond with a reduced observer which would be of order four and fully coupled.

The equations of motion are not derived in this paper. However, the physical data are given as follows: masses (m, M), rotor stiffness (k_R) and negative magnetic bearing stiffness (k_S) due to the pre-magnetization current:

$$m = 1\,\mathrm{kg} \qquad k_R = \frac{EJ}{L^3} = 206\,\mathrm{kN/m}$$

$$M = 5\,\mathrm{kg} \qquad k_S = -150\,\mathrm{kN/m}$$

To give an idea of the physical behaviour of this system the time-continuous eigenvalues are given below (the transmission zeroes are not listed here). Note that the open loop system is unstable due to the negative bearing stiffness k_S.

$$\lambda_{1,2} = \pm 35 + 0\,i \ \mathrm{Hz} \qquad \lambda_{3,4} = \pm 62 + 0\,i \ \mathrm{Hz} \qquad \lambda_{5,6} = 0 \pm 139\,i \ \mathrm{Hz}$$

The time-discrete system equations (1, 2) for this example are obtained via the transition matrix and its integral. The sample rate T is chosen to correspond with a sample frequency about 5 to 10 times higher than the highest system frequency. In this example T equals 1 millisecond.

For magnetic bearing applications it is important that a certain closed-loop static bearing stiffness k is achieved. Typically, k lies in the range of the absolute value of k_S. Thus, the three coefficients d_0, d_1, d_2 of each decentralized controller are not independent (see paragraph 2.2.). For the given example (fig. 3.) the static bearing stiffness parameter interdependence can be formulated as follows:

$$d_0 + \frac{d_1}{1 - d_2} + k - k_S = 0 \tag{9}$$

4.1. Quasi-Continuous Determination of the Feedback Parameters

The traditional approach to determine the values of the feedback coefficients is by a digital approximation of well-known continuous control laws (/7/, /8/). Very often found in magnetic bearing applications is the PD-algorithm which corresponds with a spring-damper element in classical rotor dynamics. For this case the digital first-order causal approximation of the corresponding transfer function G(s) leads to the following discrete controller transfer function G(z):

$$G(s) = P + D\,s \quad \Leftrightarrow \quad G(z) = d_0 + \frac{d_1 z^{-1}}{1 - d_2 z^{-1}} = P + \frac{D}{T}(1 - z^{-1}) \tag{10}$$

Note that d_2 is set to zero by this quasi-continuous approach, and that therefore only one feedback coefficient is freely choosable after selection of the static bearing stiffness k (9).

In the following figure k is chosen to equal the absolute value of k_S. The closed-loop step response to a unit force applied from outside in only one bearing (asymmetric load case) is simulated for the first 150 milliseconds. The rotor displacements in each bearing (x_1, x_2) and the corresponding bearing forces (u_1, u_2) are plotted for three different controller layouts respecting (9, 10). Note that one bearing force has to be twice the unit force due to the negative bearing stiffness k_S.

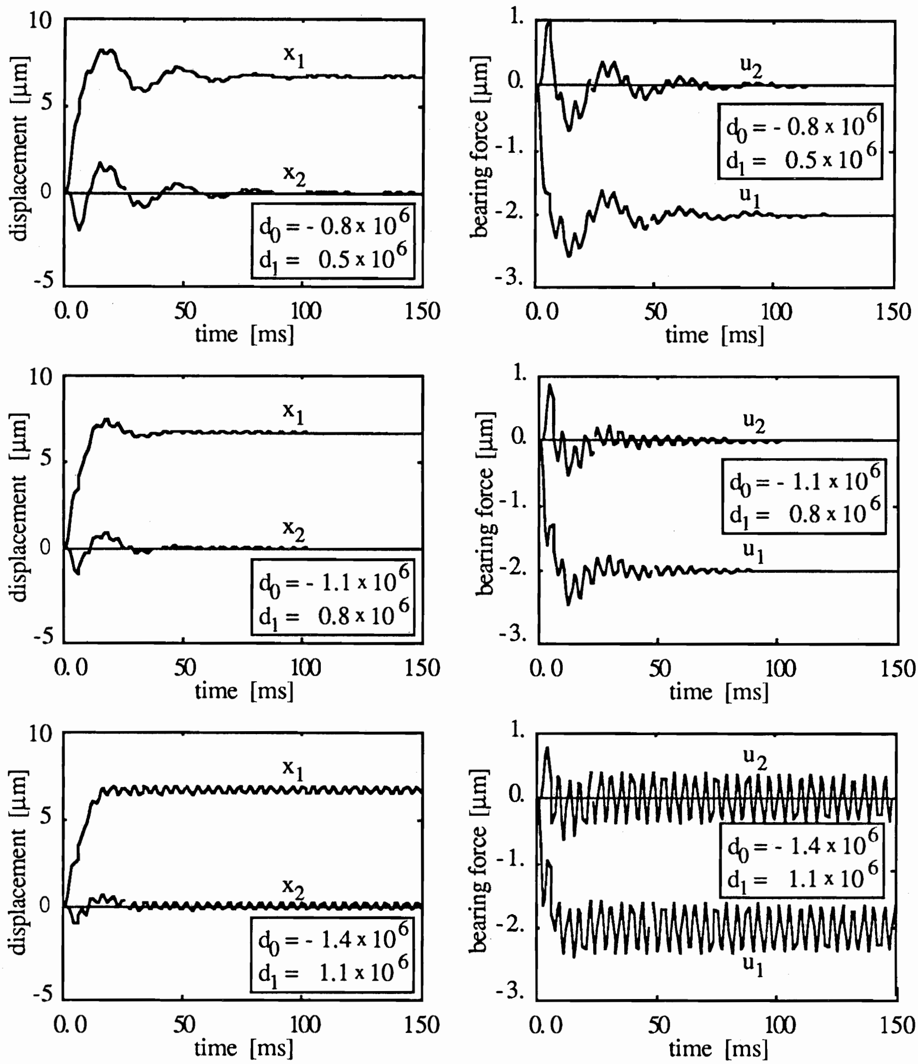

fig. 4. simulations of the step response for a controller layout based on a quasi-continuous approach

Fig. 4. shows the very interesting effect that for "low damping" (d_1 small) neither the low frequency rigid body modes nor the high frequency elastic mode are influenced in a satisfactory way. Applying "more damping", i.e. changing the one only free parameter, the rigid body mode will tend towards a good behaviour whereas the elastic mode, however, tends to get unstable.

Thus it is not possible to achieve a satisfactory closed-loop system performance using a first-order decentralized controller with a given static bearing stiffness layed out by a quasi-continuous approximation of a PD-controller.

4.2. SPOC-D Optimized Feedback Parameters

Much better results are obtained by application of SPOC-D. The results of the parameter optimization underlying the constraint of the static bearing stiffness differ in a very crucial way from those given above: the feedback coefficient d_2, set to zero by the traditional approach, turns out to play an important role in improving the system performance. Fig. 5. shows the step response for the same conditions described above but for the SPOC-D optimized controller.

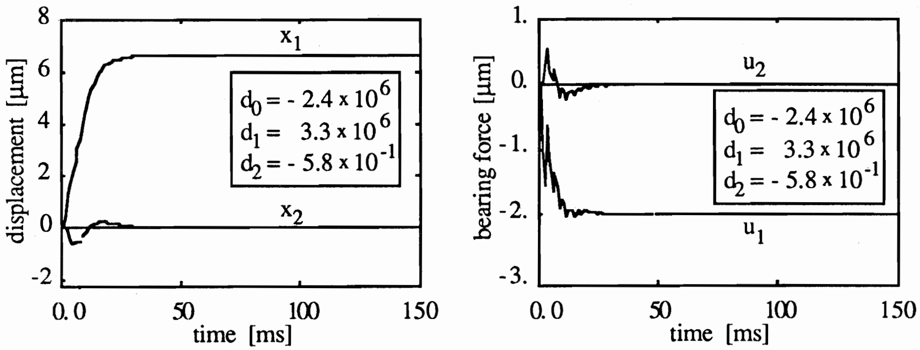

fig. 5. simulations of the step response for a controller layout based on SPOC-D

The robustness of the optimized closed-loop system with respect to changing plant parameters is not investigated in detail here. However, a comparison of the simulation results between the quasi-continuous layout and the optimized controller is shown in fig. 6. for some altered values of the mass M *but for unchanged controller coefficients*. Only the displacement x_1 is plotted.

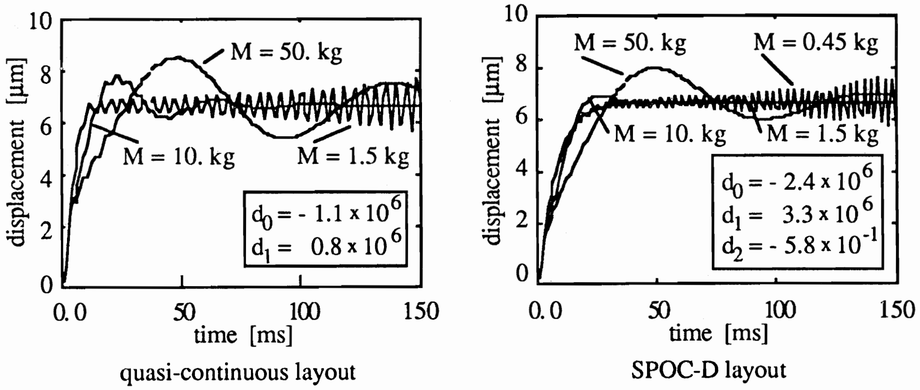

fig. 6. comparison of the step responses for different values of M

5. CONCLUSION

The layout of digital controllers with predefined structure is often based on a quasi-continuous approach. It can be shown, however, that the time-discrete approximation of a "good" time-continuous control algorithm does not always lead to comparably "good" results.

An optimization of the controller coefficients directly in the time-discrete state space leads to much better results. A reason therefore is the fact that the control parameters are optimized based on a performance index including the plant characteristics; thus the optimized controller is better adapted to the specific plant and is less sensitive to changing plant parameters.

The proposed way of optimization is very flexible. A large number of well posed problems with any kind of control structure predefinition can be handled. Any constraints of the controller coefficients, linear or nonlinear, can be considered in the necessary equations for optimality. The choice of the weighting matrices and factors used in the performance index is not problematic if the system equations are brought into a suitable normal form. A certain amount of experience, however, is necessary to find good solutions in a short time.

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