Modeling for Flexible Mechanical Systems

O. Matsushita*, H Bleuler**, T. Sugaya*, R. Kaneko***

* Mechanical Engineering Research Laboratory, Hitachi Ltd., 502 Kandatsu-machi, Tsuchiura-shi, Ibaraki, Japan 300

** Institute of Mechanics, Swiss Federal Institute of Technology, CH 8092 Zürich, Switzerland

*** Hitachi Works, Hitachi Ltd., 1-1 Saiwaicho 3 chome, Hitachi-shi, Ibaraki, Japan 317

Outline

An alternative modelling technique is proposed in place of the well known modal reduction method for mechanical system modelling. The method, called **quasi-modal reduction**, belongs to the category of mode synthesis methods and represents a special way of substructuring.

Its main advantage is the **adaptability** of the reduced mechanical model to varying boundary conditions, as they are often encountered in control design for mechatronics systems. A mass-spring model of the reduced mechanical system can be derived for flexible mechanical systems.

As an application example, a possible control layout method, a Luenberger observer modified for mechanical vibrational systems, is indicated.

1. Introduction

The quasi-modal method will be presented with an example of a flexible rotor system supported by a contact-free active electromagnetic bearing. The method is suited for modelling and control layout of a quite general class of mechanical vibrational systems.

For this purpose, the FEM equations commonly used for vibration simulations must be simplified. The major drawback of the modal method is the inaccuracy of the results for varying boundary conditions, as the mode shapes depend themselves on a certain boundary condition. The controller in the process of being designed determines these boundary conditions and hence the mode shapes.

The method proposed here uses two types of modes independent of the actual boundary conditions: The modes for constrained (i.e. fixed) boundary conditions are combined with deflection modes generated by unit inputs at the boundary conditions.

2. The Quasi-Modal Method Demonstrated with a Simple Example

2.1 FEM-Model and Equations of Motion

A simple flexible rotor is supported by an ideal hinge and a magnetic bearing as shown in Fig. 1:

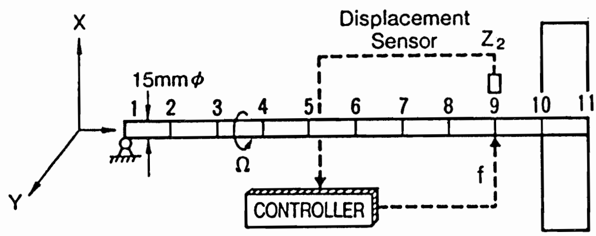

Fig. 1: Flexible rotor example. The shaft vibration is measured with a gap sensor placed at the magnetic bearing (collocation). The FEM-model uses the 10 shaft elements shown. This 40-degrees-of-freedom model will be used as the reference model.

The equations of motion are derived from the finite element model. The complex notation z = x + iy (with the radial directions x and y) is applied, as usual in rotor dynamics.

The displacement vector z is divided into "external" displacements z_2 and "internal" displacements z_1. "External" degrees of freedom are at bearing and sensor locations, "internal" ones are not accessible. The equations of motion can then be represented as follows:

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{bmatrix} + \begin{bmatrix} K_{11} K_{12} \\ K_{21} K_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} f(t) \tag{1}$$

with the diagonal mass matrix, the stiffness matrix structured as shown and the bearing force f(t). The example, simple as it is, contains the important features of an actual flexible mechanical system. The coordinate z_2 acts as a "window" giving access to the mechanical system (Fig. 2) :

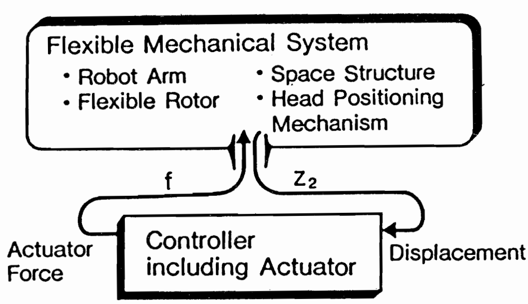

Fig. 2: The measured mechanical variables z_2 and the actuator forces f(t) act like an access "window" to a mechanical system

The FEM program used for the reference model is the HIROT-package /1/ based on the stiffness matrix method. For an actual machine rotor, the number of degrees of freedom of the FEM-model often becomes so large, that it is not suitable even for advanced control layout methods. Therefore, a reasonably reduced model is sought, which still accurately describes the first few vibration modes.

2.2 Modal Reduction

The well known modal reduction method is based on mode shapes. The mode shapes depend on the reaction force f(t). This reaction force on the right hand side of equation (1) is replaced by a pure stiffness K_b in the bearing, as produced e.g. from displacement feedback:

$$f(t) = - K_b \, z_2(t) \qquad (2)$$

This yields a homogeneous conservative equation. The eigenfrequencies (Fig.3) are computed in function of bearing stiffness K_b. For modal decomposition, a fixed and realistic value of K_b is selected, e.g. 30 DN/mm. The number of corresponding mode shapes V_i is truncated arbitrarily. The transformation for the first three modes is:

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \sum_{i=1}^{3} \begin{bmatrix} V_{i1} \\ V_{i2} \end{bmatrix} S_i \qquad (3)$$

with the modal coordinates S_i and the mode shape vectors V_i structured as the z-vectors in equation (1) . The modal equations of motion are:

$$M^* \ddot{S} + K^* S = R_b f(t) \quad \text{where } M^* = \text{diag}(m_i^*)\,,\ K^* = \text{diag}(k_i^*) \text{ and } R_b = \begin{bmatrix} V_{12} \\ V_{22} \\ V_{32} \end{bmatrix}$$

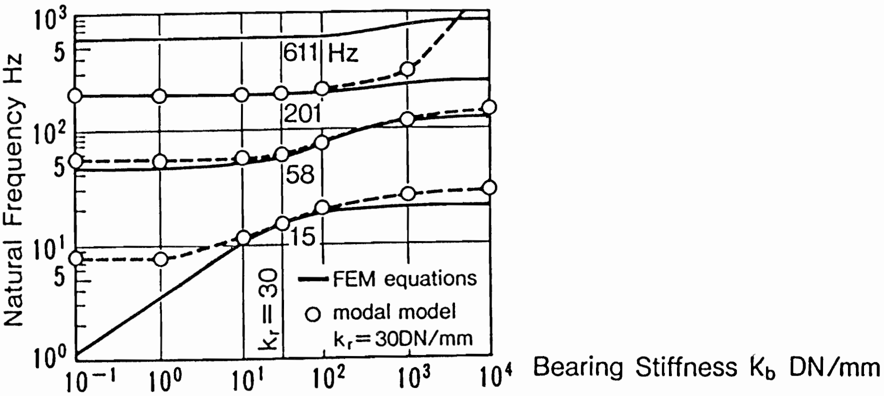

Fig. 3 Eigenfrequencies of the FEM model and modal reduced system of the rotor example in function of bearing stiffness K_b with the limit cases for free and fixed boundary condition for small resp. large K_b .

Numerical values for the example of Fig. 1 are:

$$m_i^* = \begin{bmatrix} 13.5 \\ 7.6 \\ 0.3 \end{bmatrix}, \quad \omega_i = \begin{bmatrix} 15 \\ 58 \\ 201 \end{bmatrix} \text{Hz where } k_i^* = m_i^* \omega_i^2 \quad \text{and } R_b = \begin{bmatrix} .47 \\ .8 \\ .23 \end{bmatrix}$$

In fig. 3, the eigenfrequencies of this reduced model are plotted in function of the bearing stiffness, as was done for the FEM reference model. There is exact agreement only for the selected value of the bearing stiffness. The large deviations (specially of the first mode) are an incentive to look for a more suitable reduction method than modal truncation.

2.3 Quasi-modal Reduction

In analogy to mode synthesis /2/ and in accordance with our earlier publications /3/, a different reduction matrix is proposed here. The aim is a reduced model, that can be used for various bearing characteristics. Therefore, the method should be independent of boundary conditions (stiffness or damping) at the actuators, i.e. from controller parameters. This is achieved by separating the rotor (inner system) from the bearing to be designed (outer system.).

The **inner system** is characterized by the mode shapes with fixed boundary conditions, as shown in Fig. 4a. These modes, independent of the bearing parameters, are called Φ_i .

For the connection with the outside, i.e. to provide access to the relevant bearing variables (displacement and force), mode shapes obtained from unit force displacement at the bearing have to be used. The input- and output matrices then contain only ones (at the bearings) and zeros (elsewhere). These modes are called δ_i. For our example with only one variable z_2, there is only one δ-mode, the rigid-body mode of Fig. 4b, equal to the free boundary condition first mode.

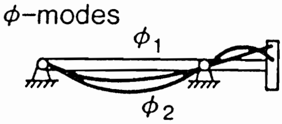

Fig. 4a Inner system mode shapes (only two shown), eigenfrequ. ω_{pi}

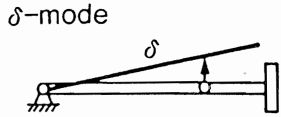

Fig. 4b Deflection mode shape

Fig. 4 Mode shapes for the quasimodal transformation.

The transformation matrix is analogous to the modal method, but now using the Φ- and δ-modes. A reduced system of 3rd order is obtained by using the δ-mode and two Φ-modes in this way:

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \delta \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ z_2 \end{bmatrix} \quad (4)$$

In opposite to the modal reduction, physical bearing displacements z_2 remain in the variable vector of the reduced system. Thus, the controller is easily connected to the mechanical system. The quasi-modal reduced system has the following general structure:

$$M^* \begin{bmatrix} \ddot{s} \\ \ddot{z}_2 \end{bmatrix} + K^* \begin{bmatrix} s \\ z_2 \end{bmatrix} = R_b f(t)$$

The system reduced to 3rd order is:

$$\begin{bmatrix} m_1^* & 0 & m_{c1} \\ 0 & m_2^* & m_{c2} \\ m_{c1} & m_{c2} & m_\delta \end{bmatrix} \begin{bmatrix} \ddot{s}_1 \\ \ddot{s}_2 \\ \ddot{z}_2 \end{bmatrix} + \begin{bmatrix} k_1^* & & \\ & k_2^* & \\ & & 0 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f(t)$$

where $M^* = \begin{bmatrix} 16.7 & 0 & 16.2 \\ . & 8.08 & -4.8 \\ \text{sym} & . & 19.6 \end{bmatrix}$ $k_i^* = m_i^* \omega_{pi}^2$

$\omega_{p1}^2 = 22\,\text{Hz}$ $\omega_{p2}^2 = 126\,\text{Hz}$

The obtained reduced matrices are not diagonal. Due to the orthogonality of the Φ-modes, however, they are partially diagonal. Due to the simple structure of the input- (and output-) matrix, the bearing stiffness can easily be included or excluded in matrix K*. This is an advantage over the modal transformation.

The most important advantage of the quasi-modal reduction is the good agreement of the eigenvalues of the reduced system and FEM-model for the full range of the stiffness K_b (fig.5) . The agreement is best for the lowest modes, as seen clearly when comparing fig. 3 and fig. 5.

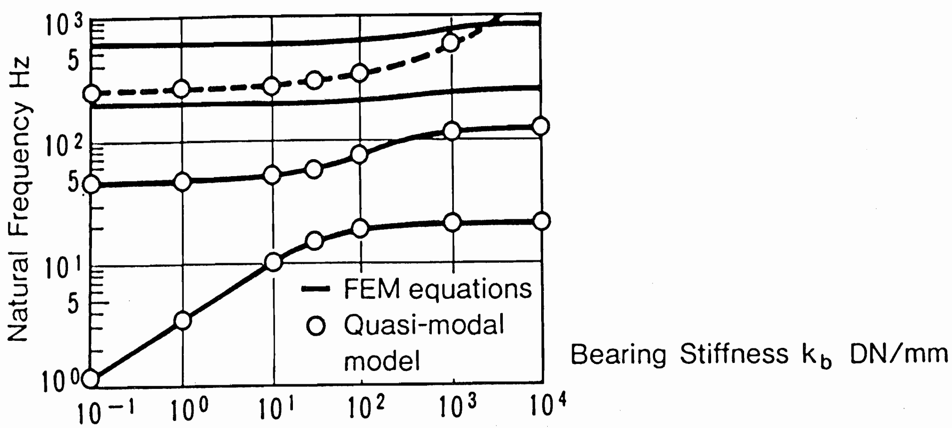

Fig. 5 Eigenfrequencies of the FEM model and quasi-modal reduced system of the rotor example in function of bearing stiffness K_b with the limit cases for free and fixed boundary condition for small resp. large K_b .

The same advantage of better accuracy of the low modes is also present for the modal damping ratios in function of bearing damping (i.e. velocity feedback). This is shown in fig. 6:

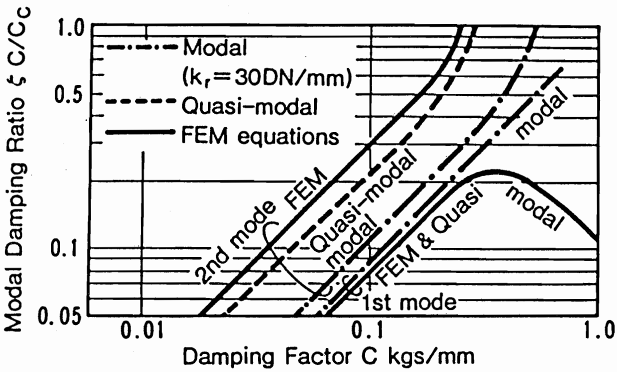

Fig. 6 Modal damping of the FEM model and the two reduced systems in function of bearing damping (and constant stiffness K_b = 30 DN/mm). **The damping ratio saturation, important for bearing controller design, is present in the FEM model and the quasi-modal reduced system, but not in the modal reduced one.**

The superior accuracy of the quasi-modal reduced system in function of widely varying bearing parameters is due to the use of the combination of Φ-modes and δ-modes.

3. Equivalent Quasi-modal System

3.1 Why the equivalent transformation?

The quasi-modal reduced system as presented in the previous chapter is characterized by a non-diagonal mass matrix, i.e. mass coupling, and a diagonal stiffness matrix. In some cases, a physically better understandable structure with stiffness coupling and diagonal mass-matrix is preferred.

The displacement vector of the quasi-modal system contains the time-variable weighting values S_i for the bending modes. These variables must be interpreted as relative displacements. Absolute displacements z^*_i are introduced for each bending mode:

$$S_i = a_i (z^*_i - z_2) \qquad (5)$$

A diagonal mass matrix is obtained, by applying this transformation to the quasi-modal system and defining the constants a_i as : $a_i = m_{ci} / m^*_i$.

The structure of the transformed system (shown only for third order) is:

$$\begin{bmatrix} m_{eq1} & 0 & 0 \\ 0 & m_{eq2} & 0 \\ 0 & 0 & m_{eq\delta} \end{bmatrix} \ddot{\begin{bmatrix} z_1^* \\ z_2^* \\ z_2 \end{bmatrix}} + \begin{bmatrix} k_{eq1} & & -k_{eq1} \\ & k_{eq2} & -k_{eq2} \\ sym & & k_{eq\delta} \end{bmatrix} \begin{bmatrix} z_1^* \\ z_2^* \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f(t) \qquad (6)$$

where $M_{eq} = \begin{bmatrix} 15.7 & \cdot & \cdot \\ \cdot & 2.85 & \cdot \\ \cdot & \cdot & 25.8 \end{bmatrix}$, $k_{eqi} = m_{eqi}\,\omega_{pi}^2$, $k_{eq\delta} = \sum_i k_{eqi}$

This system can be interpreted directly in the physical mass-spring system shown in fig. 7, along with a summary of the calculation procedure.

Coupling factor $a_i = \dfrac{m_{ci}}{m_i^*} = \dfrac{\phi_i^t M \delta}{\phi_i^t M \phi_i}$

Equivalent mass $m_{eqi} = a_i^2 m_i^*$

Equivalent residual mass $m_{eq\delta} = m_\delta - \Sigma m_{eqi}$

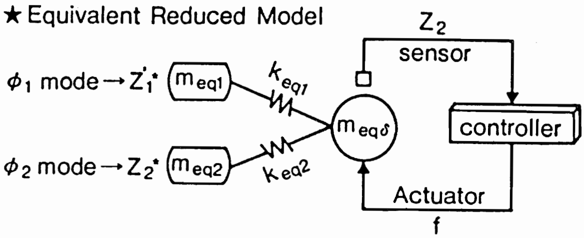

Fig. 7 Equivalent quasi-modal reduced system and its physical interpretation.(only two bending modes shown)

3.2 Example of a model with two bearings

In this discussion, a most simple example has been used. It is however a quite general procedure, not restricted to collocation or only one sensor-actuator. A slightly more general example, with two bearings and three bending modes in the reduced model (5 degrees of freedom per radial direction), is shown in fig. 8 and fig. 9 . A gyroscopic matrix can also easily be included.

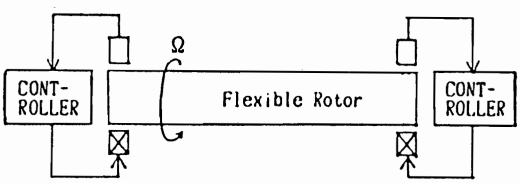

Fig. 8 Example of a rotor with two radial bearings and sensors.

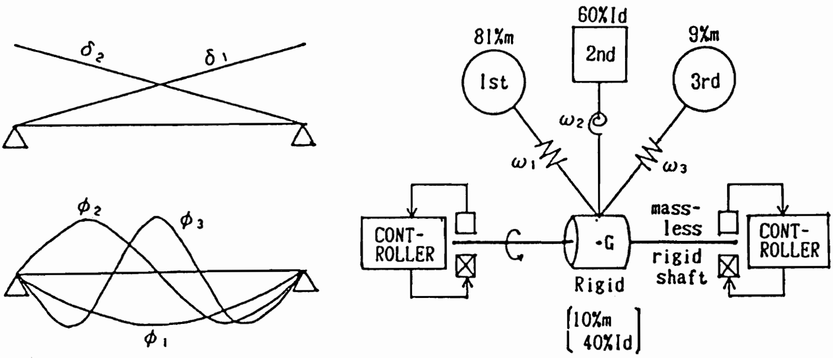

Fig.9 Equivalent reduced model for the rotor with two bearings of fig.8. and five degrees of freedom per radial direction.

4. Transfer Function

The quasi-modal reduced system can be represented in form of a transfer function relating bearing-force as input variable with bearing displacement as output variable. This is shown in fig. 10 for the simple rotor example of fig. 1 and the corresponding reduced model.

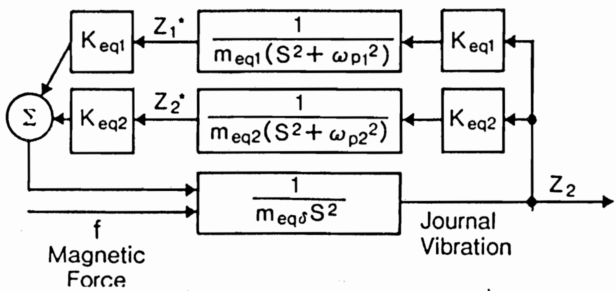

Fig.10 Block diagram of the transfer function of the equivalent quasi-modal reduced system for the example of fig. 1 and 7 . It is based on equations (6) ; the variable s is the Laplace domain variable.

The block diagram of fig. 10 can be reduced to the following transfer function, using the parameter definitions given with equ. (6) and fig. 7:

$$G(s) = \frac{z_2}{f} = \frac{(s^2+\omega_{p1}^2)(s^2+\omega_{p2}^2)}{m_{eq\delta}\, s^2 (s^2+\omega_{f1}^2)(s^2+\omega_{f2}^2)} \qquad (7)$$

where ω_{f1} and ω_{f2} are natural frequencies of the system with free boundaries (no actuators)

$m_{eq\delta}$ is the equivalent mass for the main rigid body

ω_{p1} and ω_{p2} are natural frequencies of the inner system restricted by the pin boundaries,

Note that this transfer function reflects the correct behavior of the eigenfrequencies when the bearing stiffness moves from free to pinned boundary condition.

5. Luenberger Observer Based Bearing control

This control scheme basically consists of state-feedback combined with a model of the mechanical system used to estimate ("observer") the unmeasured state variables in real-time. The description given in control theory textbooks is usually in state-base description. Here, the equations of motion notation is used in order to provide some additional physical insight into the observer dynamics. A complete system is shown in fig. 11:

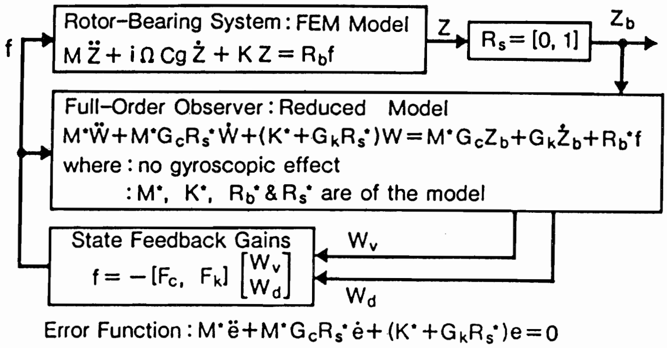

Fig. 11 Closed-loop mechanical system with observer-based state feedback. The observer dynamics is determined by matrices Gc and Gk which can be interpreted as stiffness and damping respectively

6. Conclusion

The proposed quasi-modal reduction method overcomes some clear disadvantages of the modal truncation reduction method, namely its dependence on boundary conditions.

The higher effectiveness, accuracy and easier physical interpretation of the quasi-modal method are demonstrated and are now being tested for the control layout of vibrational systems like active magnetic bearing support of elastic structures.

7. References

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