

Balancing Measurement System Using Magnetic Bearings

Toshiro Higuchi

Institute of Industrial Science, University of Tokyo, Tokyo

Takeshi Mizuno

Faculty of Engineering, Saitama University, Urawa

summary

The principles and features of a measurement system with magnetic bearings for providing information for balancing a rotor are described. The proposed system uses magnetic bearings not only as a device supporting a rotor but also as a device measuring and compensating unbalance forces. In the system the total forces acting on a rotor can be balanced by controlling the bearing forces actively; measuring and locating unbalance on the rotor are performed in such states. Some experiments of measurement are carried out by using a magnetic bearing spindle as a single-plane balancing machine. The obtained results show the feasibility of a measurement system for balancing a rotor based upon the proposed principles of measuring.

1 Introduction

The magnetic bearing can suspend a rotor without any mechanical contact and lubrication. Since an active magnetic bearing can control force acting on the rotor dynamically, it can have additional functions which have not been achieved by other bearings. A control method, by which a magnetic bearing can suspend an unbalanced rotor without whirling, has been presented based upon the theory of output regulation with internal stability¹⁾. In the designed control system centrifugal forces due to unbalance are estimated by an observer and cancelled by electromagnetic force during rotation. When the position of a rotor is regulated to rotate without whirling, the total forces acting on the rotor are balanced; information which is necessary to balance the rotor can be obtained in a direct way from the controlled electromagnetic forces of the bearing. In this paper this regulation technique is applied to measuring and locating unbalance on rotor actually.

2 Description of the Test Instrument

2.1 Structure of the Instrument Figure 1 shows the section view of the test instrument used for investigations of measuring and compensating unbalance forces. It consists of an axial magnetic bearing, two radial magnetic bearings (a, b), a motor-stator and a rotor. The rotor, to which an object of balancing will be attached, is suspended by the magnetic bearings. A radial magnetic bearing has four electromagnets which are set in four equally spaced positions in the bearing plane and an inductive-type device for detecting radial movements of the rotor. The motor is three-phase induction-type and driven by an inverter which can vary the supply frequency.

2.2 Dynamic Modelling The attractive force of each magnet of the radial bearings, which is represented as F_n ($n=1, \dots, 8$), can be approximately given by the following equation:

$$F_n = K_n (I_n / D_n)^2 \quad n=1, \dots, 8 \quad (1)$$

where

K_n : coefficient of the magnet

I_n : excited current of the magnet ($= \bar{I}_n + i_n$)

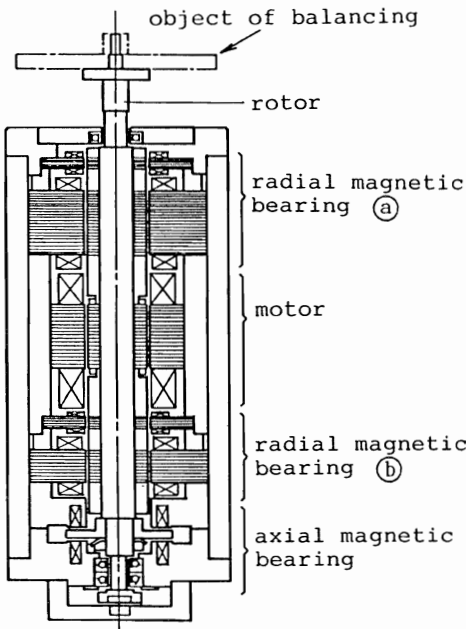


Fig. 1 Structure of experimental setup

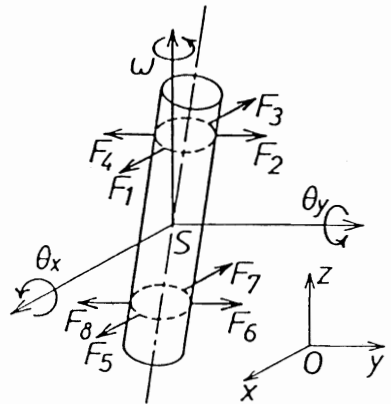


Fig. 2 Coordinate axes and forces acting on the rotor

\bar{I}_n, i_n : stationary and incremental components of I_n
 D_n : gap between rotor and pole of the magnet ($=\bar{D}_n+d_n$)
 \bar{D}_n, d_n : stationary and incremental components of D_n

For small motions about the stationary, the magnetic force can be approximated by the linear relation:

$$F_n = \bar{F}_n - G_n d_n + H_n i_n \quad n=1, \dots, 8 \quad (2)$$

where

$$\bar{F}_n = K_n (\bar{I}_n / \bar{D}_n)^2, \quad G_n = 2\bar{I}_n^2 / \bar{D}_n^3, \quad H_n = 2\bar{I}_n / \bar{D}_n^2$$

A coordinate frame O -xyz and the direction of F_n are shown in Fig.1. In nominal rotor position, the center of the rotor S is at the coordinate origin O and the center line agrees with z -axis. The stationary currents are set to realize the equilibrium conditions:

$$\bar{F}_1 - \bar{F}_3 + \bar{F}_5 - \bar{F}_7 = 0 \quad (3)$$

$$\bar{F}_2 - \bar{F}_4 + \bar{F}_6 - \bar{F}_8 = 0 \quad (4)$$

$$-(\bar{F}_2 - \bar{F}_4)l_1 + (\bar{F}_6 - \bar{F}_8)l_2 = 0 \quad (5)$$

$$(\bar{F}_1 - \bar{F}_3)l_1 - (\bar{F}_5 - \bar{F}_7)l_2 = 0 \quad (6)$$

and decoupling conditions:

$$(G_1 + G_3)l_1 - (G_5 + G_7)l_2 = 0 \quad (7)$$

$$(G_2 + G_4)l_1 - (G_6 + G_8)l_2 = 0 \quad (8)$$

where

l_1, l_2 : distance between the center of the rotor S and the magnets of the radial bearing (a) and (b)

Then the equations of motion are given by

$$m\ddot{x}_S - (G_1 + G_3 + G_5 + G_7)x_S = H_1 i_1 - H_3 i_3 + H_5 i_5 - H_7 i_7 + m\epsilon\omega^2 \cos(\omega t + \alpha) \quad (9)$$

$$m\ddot{y}_S - (G_2 + G_4 + G_6 + G_8)y_S = H_2 i_2 - H_4 i_4 + H_6 i_6 - H_8 i_8 + m\epsilon\omega^2 \sin(\omega t + \alpha) \quad (10)$$

$$I_R \ddot{\theta}_x + I_a \omega \dot{\theta}_y - ((G_2 + G_4)l_1^2 + (G_6 + G_8)l_2^2)\theta_x - (H_2 i_2 - H_4 i_4)l_1 + (H_6 i_6 - H_8 i_8)l_2 + (I_R - I_a) \tau \omega^2 \cos(\omega t + \beta) \quad (11)$$

$$I_R \ddot{\theta}_y - I_a \omega \dot{\theta}_x - ((G_1 + G_3)l_1^2 + (G_5 + G_7)l_2^2)\theta_y + (H_1 i_1 - H_3 i_3)l_1 - (H_5 i_5 - H_7 i_7)l_2 + (I_R - I_a) \tau \omega^2 \sin(\omega t + \beta) \quad (12)$$

where

x_S, y_S : displacements of the rotor in x and y directions

θ_x, θ_y : angular displacements of rotor axis about x and y axes

m : mass of the rotor

I_a, I_r : polar and transverse mass moments of inertia of the rotor

α, β : parameters on angular location of static and dynamic unbalance

ϵ, τ : amount of static and dynamic unbalance

ω : angular velocity of the rotor

From eqs.(9),..., (12) the whole system can be divided to a subsystem related to translation and a subsystem related to rotation. As a result the dynamics of the magnetic bearing system is expressed by a set of equations of the type:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{D}\mathbf{w}(t) \quad (13)$$

$$\dot{\mathbf{w}}(t) = \mathbf{E}\mathbf{w}(t) \quad (14)$$

where

$$\mathbf{x} = [x_1 \quad \dot{x}_1 \quad x_2 \quad \dot{x}_2]^t, \quad \mathbf{u} = [u_1 \quad u_2]^t, \quad \mathbf{w} = [w_1 \quad w_2]^t$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a & 0 & 0 & c\omega \\ 0 & 0 & 0 & 1 \\ 0 & -c\omega & a & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ b & 0 \\ 0 & 0 \\ 0 & b \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}$$

3. Compensation and Measurement of Rotor Unbalance

3.1 Compensation of Unbalance Forces The measuring method proposed in this paper are based upon a control system developed by the authors¹⁾. The ways of designing the control system are

- (1) The system is stabilized by a state feedback since an attractive magnetic bearing system is inherently unstable without control.
- (2) Since centrifugal forces due to unbalance cannot be detected in an usual magnetic bearing system, an observer is constructed in order to estimate unbalance forces.
- (3) Using the output of the observer the effects of rotor unbalance are cancelled by controlling incremental currents of the electromagnets.

The resultant control input is

$$\mathbf{u}(t) = \mathbf{P}_1 \mathbf{x}(t) - \hat{\mathbf{w}}(t) / b \quad (15)$$

where \mathbf{P}_1 is selected to stabilize a closed-loop system:

$$\dot{\mathbf{x}}(t) = (\mathbf{A} + \mathbf{B}\mathbf{P}_1) \mathbf{x}(t) \quad (16)$$

and $\hat{\mathbf{w}}(t)$ is the output of the observer which converges to $\mathbf{w}(t)$:

$$\lim_{t \rightarrow \infty} \hat{w}(t) = w(t) \quad (17)$$

Then it is concluded that

$$\lim_{t \rightarrow \infty} x(t) = 0 \quad (18)$$

It means that whirling motions due to unbalance disappear. The details are described in ref.1.

3.2 Measurement of Rotor Unbalance When the position of the rotor is regulated, the bearing forces are cancelling the unbalance forces. Necessary data for balancing the rotor, can be obtained in a direct way from the resultant bearing forces. This measuring method is to be said as a dynamic measurement with a closed-loop system, while a common method of measuring for balancing is a measurement with an open-loop system.

In an usual centrifugal balancing machine, measurements are performed at a speed far below or far above the resonant frequency of the rotor support system because it is difficult to measure accurately near the resonant frequency. In contrast with common balancing machines, test speed can be selected independently of the natural frequency of the machine in a proposed measurement system. Therefore it is possible to balance a rotor at its actual operation speed.

4. Experiments

4.1 Methods Figure 3 shows a photograph of the test instrument; its section view has been shown in Chapter 2. This instrument, which was designed as a spindle for grinding test, is used as a single-plane vertical-type balancing machine.

A test-object of balancing which is shown in Fig.4, is concentrically mounted to the head of the suspended rotor. A weight can be attached to the object at equally spaced positions in a face of the object for introduction of known unbalance. The angular position of unbalance is measured counter-clockwise from the reference position which is pointed by an arrow in Fig.4. A reflective-type photo-sensor detects a black mark which is pasted at the reference position and generates a reference signal.

As is mentioned above, when the position of the rotor is regulated, the bearing forces are identical with corrective

forces to balance total forces acting on the rotor. For a direct measurement of bearing force, however, proper sensing devices, as piezoelectric or strain-gage transducers, must be incorporated in the instrument. On the other hand an incremental component of the current is proportional to the corrective force(see eq.(2)) and can be measured without special sensing devices. Thus current monitoring is used to determine the amplitude and location of unbalance in the experiments.

4.2 Results. The stationary motions of a rotating unbalanced rotor are shown in Fig.5; the angular velocity of the rotor is 314 rad/s. In the figures x_a and x_b refer to the outputs of the displacement sensor of the radial bearing (a). When the magnetic bearing system is only stabilized, the rotor is whirling with amplitude of about $4.5 \mu\text{m}$ (see (a)). As contrasted with the case (a), the whirling motion becomes remarkably small when the compensation for unbalance is applied (see (b)). The results show that even an unbalanced rotor behaves like a balanced rotor when it is supported by magnetic bearings with the unbalance compensation controller. The wave-

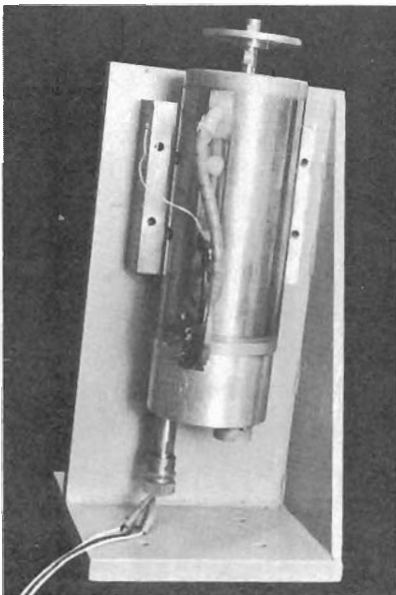


Fig.3 Outlook of experimental instrument

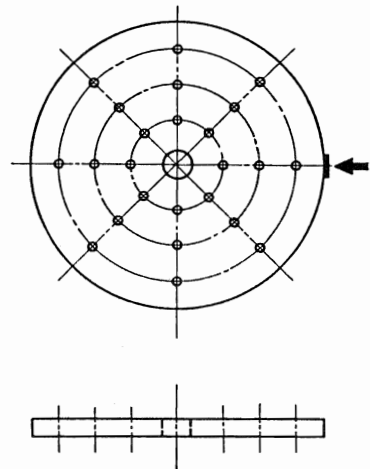
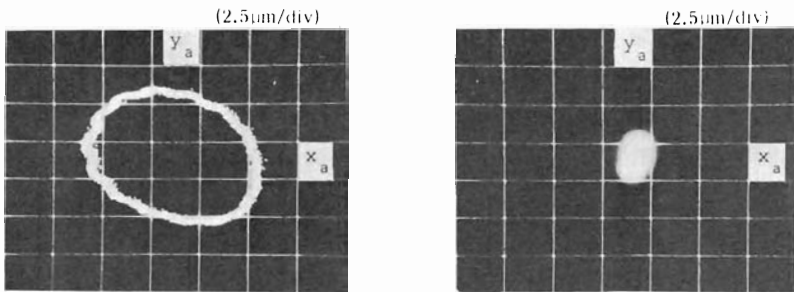


Fig.4 Example of test-object

forms of the incremental current i_3 with the compensation for unbalance applied are shown in Fig.6; a weight is attached to the test-object (see (b)) or not (see (a)). The amount and position of unbalance are determined from the amplitude and the phase relation to the reference signal of the current. The net unbalance introduced by a weight is obtained by comparing the measurement results in case(a) and (b). This procedure of measuring can reject the influences of initial unbalance of the suspended rotor and misalignment of the test-object.

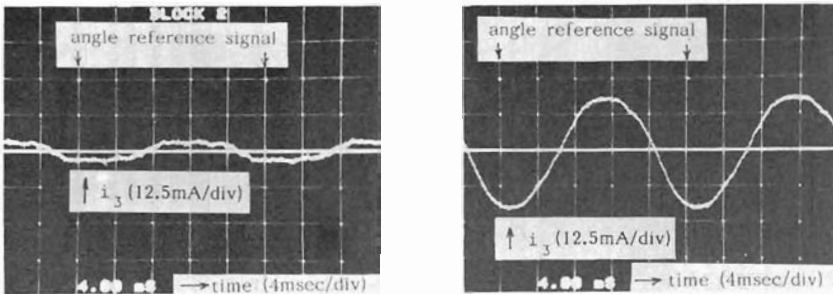
The results of measurements are shown schematically in Fig.7; the position of each circle indicates the actual or estimated location of an attached weight and its area is proportional to the mass of the weight. The actual mass of an unbalance weight is 0.25g. The rotor is driven to rotate in (a) counter-clockwise



(a) without the compensation for unbalance

(b) with the compensation for unbalance

Fig.5 Stationary motions of an unbalanced rotor suspended by the magnetic berings (scale: 2.5 $\mu\text{m}/\text{div}$)



(a) without an unbalance weight

(b) with an unbalance weight attached to the test-rotor

Fig.6 Current and angle reference signal when the compensation for unbalance works

($\omega=314\text{rad/s}$) and (b)clockwise ($\omega=-314\text{rad/s}$) directions.

It is seen from Fig.7 that the estimated amount and location coincide with the actual ones to some degree. The measured amount of unbalance is, however, about 1.6 times as great as the actual one. The main source of error is probably the parameter estimation error of H_n and can be eliminated by a proper calibration. It is also seen from Fig.7 that each estimated location tends to be shifted to the direction of the rotor rotation from the correct one. This tendency is probably due to the delay-time of the photo-sensor generating a reference signal. Using a faster-response detector or averaging the located angles which are measured in clockwise and counter-clockwise rotating, more accurate location of unbalance will be obtained.

5 Conclusions

The principles of a measurement system with magnetic bearings for providing information for performing balancing operations have been presented. The experiments demonstrate the possibility of developing a new-type balancing machine with magnetic bearings.

References

1. Mizuno, T.; Higuchi, T.: Compensation for Unbalance in Magnetic Bearing System (in Japanese). Trans. Society of Instrument and Control Engineers, 20 (1984) 1095-1101.

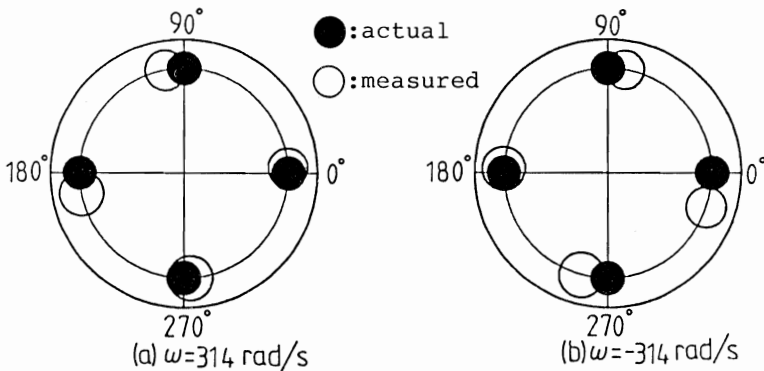


Fig.7 Results of measurements

Digital Control

