A Suppression Method of Conical Motion of an Axi-Symmetrical Spinning Rigid Rotor Suspended by Magnetic Bearings

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Summary

New definitions of nutation and precession are proposed, which are usuful not only for physical understanding but also for intuitive interpretation of the optimal control of axi-symmetrical spinning rigid bodies. At first it is shown that the spin axis does coning motion about the instantaneous angular momentum vector with constant rotational speed (nutation angular velocity) regardless of its coning angle.
Next, steady state coning motions, which have been called as steady state precessional motion, are discussed based on angular momentum vector and it is shown that they have two modes; precessional mode and nutational one.
Lastly, based on angular momentum vector, a method of control for conical motion of a spinning rigid rotor supported by a magnetic bearing system is derived and it has the same control structure as the optimal control which has been derived as a regulator problem by referenced paper.

Introduction : There were some confusion in the field of attitude dynamics and control of spacecraft at the beginning of the space age because of inadequate definitions of precession and nutation which are found in textbooks of classical dynamics. Moreover, there is no clear interpretation of dynamical behavior in the field of magnetic bearings. In this paper, new definitions of precession and nutation are proposed, which are useful not not only for physical understanding but also for intuitive interpretation of the optimal control of axi-symmetrical spinning rigid bodies. And then it is proved that the spin axis rotates about the instantaneous total angular momentum vector with constant nutational velocity regardless of its magnitude of coning motion using small cone angle approximation. Next, steady state coning motions, which have been called as steady state precessional motion, are discussed based on angular momentum vector and it is shown that they have two modes; precessional mode and nutational one. Lastly, based on angular momentum vector, a method of control for whirling motion of a spinning rigid rotor supported by a magnetic bearing system is derived and it has the same control structure as the optimal control which has been derived as a regulator problem by a referenced paper [2].

1. New Definition of Nutation and Precession, and Coning Motion

In textbooks of classical dynamics, precession is defined as horizontal motions of spinning axis, whereas nutation is defined as vertical ones. Here, the new definitions which are the same as the ones of attitude control field of spacecraft are as follows :

Precession: motion of the total angular momentum vector(H, invisible) of the rotor due to outer torques(T).

Nutation: coning motion of the spinning axis(S) about H.

We cannot see the total angular momentum vector(H), but the motion of the spinning axis(S) is nearly equal to the precessional motion when there is no nutation, i.e. $H \fallingdotseq S$. This will be shown in the next chapter.

Regardless of the both motions, a general and fundamental equation of motion is

$$dH/dt=T. \tag{1}$$

Therefore, H depends on outer torque(T) alone. On the other hand, S draws a cone about the instantaneous axis of H with constant rotational speed (nutational angular velicity)

$$h=\sigma\omega/\cos C=H/I_d \tag{2}$$

seen from inertial space, regardless of cone angle, where $\sigma=I_s/I_d$ (ratio of moment of inertia about S, I_s, to the one about the diametrical axis, I_d), ω is S-component of spinning velocity, and C is a half of cone angle. This coning motion of S in the case where T is restoring torque which is in proportion to the small tilting angle of S, θ, will be proved at the next chapter. It is well known that Eq.(2) is constant regardless of C even when C is not small, provided that H is unchanged or T = 0.

2. Equations of Motion

In the following, equations of motion of the rotor suspended by a magnetic bearing system will be derived, where $T=K\theta$, K is restoring spring constant which has the opposite sign to the spinning top on the ground.

As an inertia coordinate system, it is supporsed that mass center of the rotor is the origin, O, a vertical upside axis is Z, and two perpendicular axes are X and Y, respectively. As an moving coordinate system which expresses the attitude of the rotor, a non-spinning one is adopted, where the origin is mass center of the rotor, two axes, x and y, which are nearly horizontal and not fixed to the rotor, are near to X and Y axes, respectively, and z axis is the spinning principal axis, S.

Small tilting angle of S is θ, which is decomposed to x- and y-component as

θ_x and θ_y. Angular velocity vector of the non-spinning coordinate is expressed as $(\theta_x', \theta_y', 0)^T$, where $()^T$ notation is matrix transposition, and ['] means time derivative of a variable. Similarly, the angular momentum vector H is expressed as $(I_d\theta_x', I_d\theta_y', I_s\omega)^T$. If there is no external torque except for restoring one, then, T is expressed as follows:

$$T = -K\ (\theta_x,\ \theta_y,\ 0)^T. \tag{3}$$

Using these expressions, Eq(1) changes to the following component equations:

$$\theta_x'' + h\theta_y' + k\theta_x = 0 \tag{4}$$

$$\theta_y'' - h\theta_x' + k\theta_y = 0 \tag{5}$$

$$\omega = \text{const.} \tag{6}$$

where $k = K/I_d$. In the following, as only x- and y-component are important, we regard θ as a complex number, and express it using underlined notation:

$$\underline{\theta} = \theta_x + i\theta_y, \qquad (i = \sqrt{(-1)}) \tag{7}$$

Therefore, Eqs.(4) and (5) are combined and change into one equation:

$$\underline{\theta}'' - ih\underline{\theta}' + k\underline{\theta} = 0 \tag{8}$$

Equation (8) was derived as Euler's equation. To prove validity of Chapter 1's hypothesis, starting from this hypothesis, let us try to derive Eq.(8) geometrically.

As shown in Fig.1, using complex number plane, suppose $\underline{H}$ be positioned small angle $\underline{m}$ to the origin 0, and $\underline{S}$ be positioned small angle $\underline{\theta}$ to 0 and small angle $\underline{c}$ to $\underline{H}$. Therefore,

$$\underline{\theta} = \underline{m} + \underline{c} \tag{9}$$

$$\underline{c} = \underline{c_0}\, e^{iht} \tag{10}$$

where $\underline{c_0}$ is an initial value of $\underline{c}$, meaning rotation of $\underline{c_0}$ about $\underline{H}$ with constant rotational speed of h. As the restoring torque is perpendicular to $\underline{\theta}$,

$$\underline{dm} = \underline{dH}/H = -iK\underline{\theta}dt/H \tag{11}$$

$$\begin{aligned} \underline{dc} &= (\underline{c_0} - \underline{dm})e^{ihdt} - \underline{c_0} \\ &\fallingdotseq (\underline{c_0} - \underline{dm})(1 + ihdt) - \underline{c_0} \\ &= i\underline{c_0}hdt - \underline{dm} \end{aligned} \tag{12}$$

$$\underline{d\theta} = \underline{dm} + \underline{dc} = i\underline{c_0}hdt. \tag{13}$$

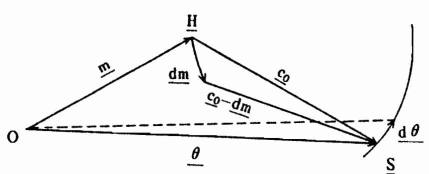

Fig. 1. Delineation for formation of differential equation

Equation (13) is a first order one, and even if we change $\underline{c_0}$ to $\underline{c}$, generality is not lost. Substituting Eq.(9) into this one, and once more differentiation gives us:

$$\underline{\theta}'' - ih\underline{\theta}' + ih\underline{m}' = 0. \tag{14}$$

Using Eqs.(11) and (2) to this one, we can get Eq.(8). Thus, the validity of Chapter 1's hypothesis has been proved.

3. Approximate General Solution of Equation of Motion

Position of S, i.e. $\underline{S}$ or $\underline{\theta}$ is expressed as approximate general solution of Eq.(8). Using subscript $_0$ as initial values, and supposing $h^2 \gg k$, $\underline{\theta}$ becomes:

$$\underline{\theta} = \underline{S} \equiv \underline{\theta}p + \underline{\theta}n \tag{15}$$

$$\underline{\theta}p \fallingdotseq (\underline{\theta}_0 + i\underline{\theta}_0'/h)e^{-i(k/h)t} \tag{16}$$

$$\underline{\theta}n \fallingdotseq (-i\underline{\theta}_0'/h)e^{i(h+(k/h))t} \tag{17}$$

From this result, general solution of $\underline{S}$ consists of a fast coning motion (nutation mode) and a slow one (precession mode). In magnetic bearings, the both modes have opposite rotational direction, whereas in spinning tops on the ground, they have the same direction because of negative value of k.

Next, let us get an expression of H motion. Radial component of H is $I_d\theta'$ whose angle expression is $I_d\theta'/H$. Phase of θ' advances to θ by $\pi/2$. Therefore, position of H on the complex number plane is $i\underline{\theta}'/h$ (see Fig.2). Differentiating Eqs.(15)~(17), we get following approximate expressions:

$$i\underline{\theta}'/h = \underline{H} - \underline{S} \fallingdotseq (k/h^2)\underline{\theta}p - (1+(k/h^2))\underline{\theta}n$$

$$\fallingdotseq -\underline{\theta}n \tag{18}$$

Adding Eq.(15) to the above expression, $\underline{H}$ is expressed as

$$\underline{H} \fallingdotseq (1+(k/h^2))\underline{\theta}p - (k/h^2)\underline{\theta}n \tag{19}$$

$$\fallingdotseq \underline{\theta}p.$$

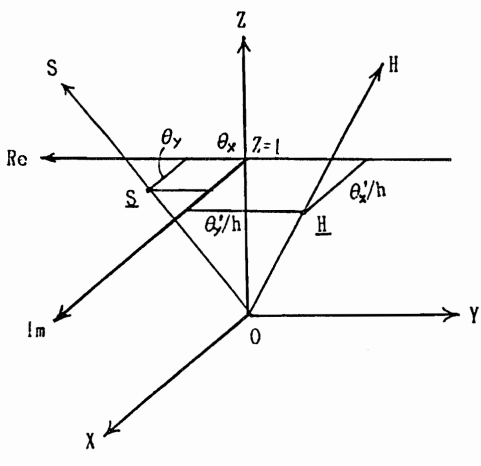

Fig. 2. Relationship between complex number plane and inertia coordinate system

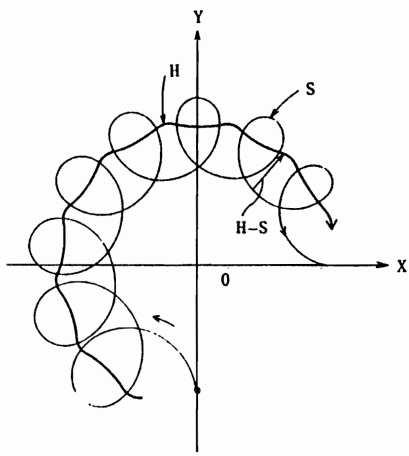

Fig. 3. General coning motion of spin axis

From the results mentioned above, $\underline{\theta p}$ expresses approximate motions of H, whereas $\underline{\theta n}$ expresses approximate radius of circular motion whose center is H. Figure 3 is a simulation example of a general case where both modes are present and are rather decaying.

4. Steady State Coning Motion

In case of spinning spacecraft, k=0, and $\underline{\theta p}$ is fastend to a constant point, and pure nutations are observed. In case of spinning tops on the ground, k≠0, and both $\underline{\theta p}$ and $\underline{\theta n}$ are present. However, at high spinning speed, $\underline{\theta n}$ damps out quickly due to friction of ground, and pure but rather decaying $\underline{\theta p}$ is observed. If there is only one mode and no damping, the motion of $\underline{S}$ is a pure circular one. Nutation is fast anti-clockwise circular motions, whereas precession is slow clockwise (anti-clockwise in case of ground spinning tops) circular ones. In textbooks of classical dynamics, either motion is treatized as fast or slow steady precessional motion.

Motion of $\underline{H}$ in the steady circular motions is described here a little. In the steady precessional motion, the second term of Eq(19) is zero, and $\underline{S}=\underline{\theta p}$, therefore, $\underline{H}$ draws a circle a bit outside (inside, in case of spinning tops) of $\underline{S}$ by (k/h^2) times of $\underline{S}$. It would be said that both $\underline{S}$ and $\underline{H}$ move getting near and parallel with each other. Fig. 4 is an example of steady precessional motion. In steady nutational motion, $\underline{H}$ consists of only the second term of Eq. (19). Therefore, $\underline{H}$ positions opposite side of $\underline{S}$ to the origin, and draws a small circle. Fig. 5 is an example of steady nutational motion.

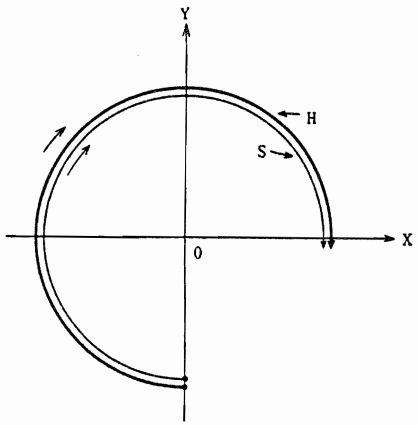

Fig. 4. Steady state precessional motion

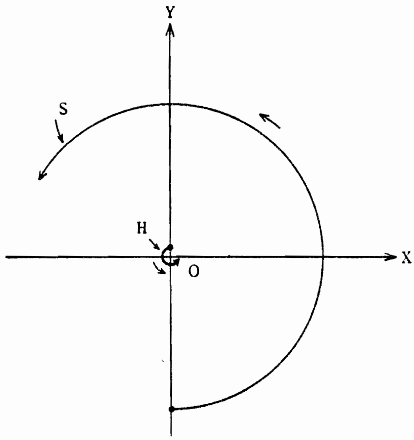

Fig. 5. Steady state nutational motion

5. Application to Suppression of Whirling Motion

Whirling motion of the rigid rotor suspended by magnetic bearings consists of two above-mentioned modes. To suppress the whirling motion, it is intuitively more easily understood to move $\underline{H}$ than to move $\underline{S}$. Because, $\underline{H}$ is governed by Eq.(1) or first order differential equation, and it has been known that $\underline{S}$ rotates about $\underline{H}$ at constant rotational speed. On the other hand, if we do not utilize directly Eq.(1), we must use the following Eq.(20) which is second order and consists of Eq.(8) and newly added control torqe T :

$$\underline{\theta}'' - ih\underline{\theta}' + k\underline{\theta} = \underline{T}/I_d \equiv \underline{u} \tag{20}$$

provided that $\underline{T}$ does not include the restoring torque. Thus, it is by far easier to move H than to try to directly move S.

For suppression of nutation, H should be moved near to S. This is a very common method in the field of attitude control of spacecraft. It is very easy to suppress precession; to bring $\underline{H}$ to the origin is sufficient. To summing up, both nutation and precession modes are suppressed at the same time, if $\underline{H}$ is brought to the origin while being moved near to $\underline{S}$. For this purpose, a concrete feedback method will be shown in the follow.

At first, to control $\underline{\theta_p}$, as is known from Eq.(16), $\underline{\theta}$ and $\underline{\theta}'$ should be fedback. On the other hand, from Eq.(18), to control nutation or $-\underline{\theta_n}$, only $\underline{\theta}'$ should be fedback. Fortunately, in both modes control, signs of θ' which should be fedback, are identical.

If we regard the magnetic bearing system as a two-gimbal servo system, each gimbal servo system has feedback loop of its own rotational position signal and its rate signal in general control. However, as $\underline{\theta}$ and $\underline{\theta}'$ are two-dimensional vectors, each gimbal servo system must include also the other one's sensor signals into its feedback loop. This is called inter-axis cross coupling feedback [1]. Returning to Eq.(16), as is already shown in Fig.2, phase of $\underline{\theta}'$ advances to $\underline{\theta}$ by $\pi/2$, or in other words, phase of complex number plane lags by $\pi/2$ to the inertia frame, therefore, $\underline{\theta}'$ should be fedback directly to each servo. On the other hand, $\underline{\theta}$ should be delayed by $\pi/2$ and then be fedback, or specifically, θ_y and $-\theta_x$ should be fedback to x and y servos, respectively. After all, rate signals should be fedback independently, and position ones in mutually coupled mode. This feedback structure is identical to the one obtained as the optimal regulator problem of the magnetic bearing systems by referenced paper [2].

6. Simulation Examples of Control

Here, supposing that control torque $\underline{U}$ includes restoring torque also, $\underline{U}$ is generally expressed as the next equation:

$$\underline{U} = -K\underline{\theta} - (K_n + K_{p1})\ \underline{\theta'} + i\,K_{p2}\underline{\theta} \qquad (21)$$

where K_n is nutation control gain, and K_{p1} and K_{p2} are precession control ones. In the following three examples, all initial conditions are the same as Fig. 3. Figure 6 is the case of approximate precession control where only the last term of Eq.(21) is used for H. Really, H approaches to the origin, but H and S diverge rapidly due to omission of K_{p1}, changing to a diverging nutation mode. Figure 7 is the case of nutation suppression where the third term of Eq.(21) is omitted (usual gimbal servo). Nutation is rapidly decayed and S and H are approaching each other, and a pure but rather decaying precession mode is observed. Figure 8 is the case where all terms of Eq.(21) are fed-back, and two modes decay rapidly at a time.

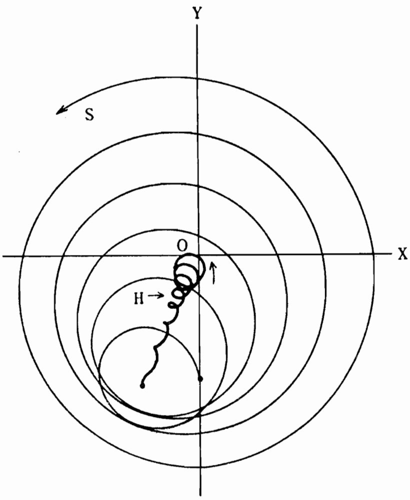

Fig. 6. Approximate precession control

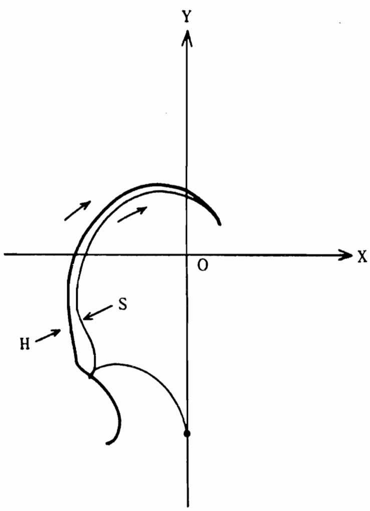

Fig. 7. Suppression of nutation alone (usual gimbal servo)

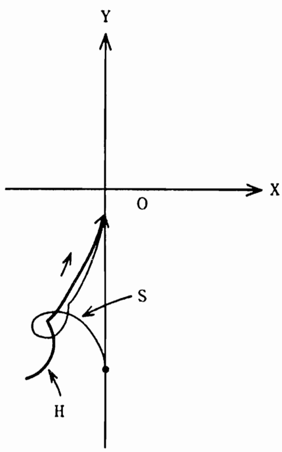

Fig. 8. Suppression of both precession and nutation

7. Conclusions.

(1) New definitions of precession and nutation are by far rational than conventional ones.

(2) Spin axis rotates about the instantaneous total angular momentum vector with a constant nutational velocity regardless of its radius of rotation.

(3) In suppression of whirling motion, it is more intuitively understandable to move the angular momentum vector than to move the spin axix directly.

(4) At high spinning velocity of the rotor, usual gimbal servo systems have little effect for suppression of precessional mode.

(5) Inter axis cross couppling feedback of angular position signals is very usufull for suppression of precessional mode because it brings the angular momentum vector to the origin.

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