Dynamic Response of Magnetically Supported Rotor

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Summary

A theoretical analysis of the dynamic response of a rotor supported in magnetic bearings is presented. This predicted response is compared with the experimental response of a small rotor subjected to an impulse loading produced by a spring loaded solenoid. Theoretical and experimental results are compared for various stiffness and damping settings for magnetic bearings.

Nomenclature

C	damping matrix	K_d	rate (derivative) gain
f	exogeneous force	K_x	position stiffness
G	control transfer function	M	mass matrix
K	stiffness matrix	X	Laplace of position
K_p	proportional gain	τ	time constant
K_i	current stiffness	α,β	transfer function coefficient

Introduction

The stability of rotors may be affected significantly by use of bearings having stiffnesses that are unequal in mutually perpendicular directions, the theory of which was presented so eloquently by Smith [1]. Credance to the work by Smith came from experiments reported in 1924 by Newkirk [2] who suggested that the unsymmetrical flexibility of the bearings was the essential feature inducing stability even in the absence of damping in the bearings. At the time of this work, no convenient methods were available to implement controllable variations in the stiffness of a bearing in two orthogonal directions although bearing supports could be designed to produce unequal, but not adjustable bearing stiffnesses.

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The use of magnetic bearings has included applications to very large machinery including high-speed multistage compressors [3]. Considerable details of the controls for magnetic bearings are now available for design and for proper applications [4]. Innumerable patents have been filed and countless papers reported in the field of magnetic bearings, some 200 of which were summarized by Humphris [5] in 1985. The reported works have probably doubled since that time as the potential for magnetic bearings has become better understood and accepted.

The option now exists through the implementation of digital controls [6] to design "smart" magnetic bearings to incorporate the findings reported decades ago. By changing the stiffnesses of the bearings in two mutually perpendicular directions as the speeds are traversed, rotors can be made more stable [7]. Simultaneously, varying the stiffnesses of magnetic bearings as a rotor speed changes offers the means to avoid running through any critical speeds as the "critical" speeds of the system will not be fixed values, but variable quantities dependent upon the controlled bearing stiffness and damping values.

As magnetic bearings become more commonplace, questions regarding some behavior details about them seem germane. This paper addresses one such detail, namely, "Can the dynamic response of a rotor in magnetic bearings be predicted theoretically when the rotor is subjected to an impulsive loading?" This is a first step in predicting the behavior of a rotor that suffers a sudden upset such as the loss of a blade in a compressor.

<u>Experimental Set-Up</u>

The arrangement used for the experimental impact testing is shown in Figure 1. The 12.7 mm (0.5 inch) diameter shaft is supported at each end by a magnetic bearing consisting of four electromagnets uniformly distributed radially about the shaft. Details of the controls and physical properties of this system have been previously reported [8]. The bearing span is 508 mm (20 in.) and a 0.8 Kg (1.8 lb) disk is located at the shaft center. A rubber coupling joins the rotor to a variable speed electric motor. Coupling properties are estimated at 17500 N/m stiffness and 8.80 N-sec/m damping. Eddy current position sensors are located vertically and horizontally at each end, displaced 30 mm from the magnetic bearing disks. Signals from these sensors are used, with the control electronics, to maintain the shaft in support and are also used as the transducers for observing the effects or results for this impact study.

A simple spring loaded solenoid, taken from an ordinary door-bell chime, is used as the impact mechanism. An electrolytic capacitor is initially charged to a given voltage.

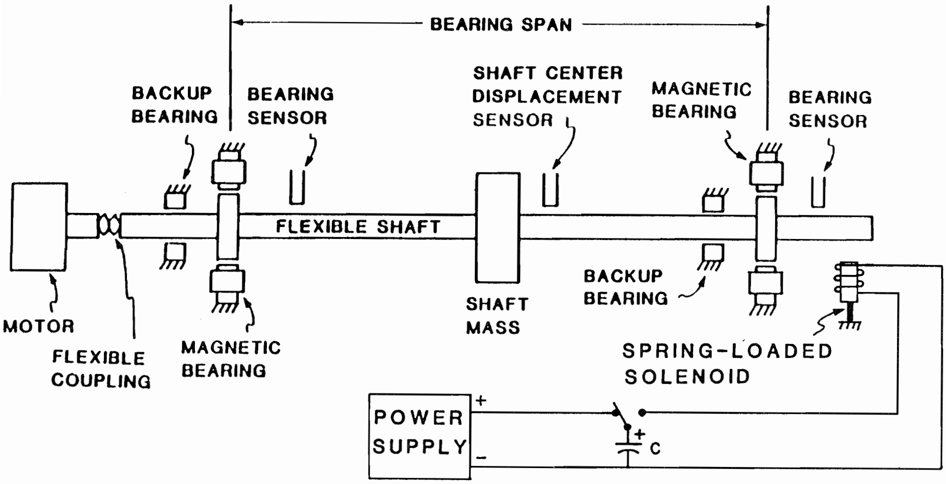

Figure 1. Experimental Schematic Diagram with Spring-Loaded Solenoid

When the push button switch is actuated, the stored charge is dumped through the spring loaded solenoid coil, causing the plunger to impact on the desired point of the shaft. The force of the impact is accurately controlled by varying the power supply voltage applied to the capacitor. Very repeatable impacts are obtained, as observed by a digital signal analyzer used to capture the transient behavior.

State Space Rotor/Bearing Model Development

To obtain a description of a rotor supported in magnetic bearings adequate for stability analysis and transient response simulation, a detailed model is required. Previously reported works have either assumed a simple second order plant model [7] or have examined higher order plant models using state space techniques, but neglected the high frequency limitations (poles) of the controller [9]. In this development, a general methodology is outlined for constructing a model which includes not only higher order plant dynamics and controller bandwidth limitations, but also permits accurate modeling of sensor/actuator collocation error. This last effect has been discussed fairly widely in the literature since 1977 [9,10].

Assuming that an undamped structure model for the rotor has been obtained of the form

$$M\ddot{\underline{x}} + K\underline{x} = \underline{f} \tag{1}$$

and that any mechanical bearings, seals, and gyroscopic effects can be modeled by linear stiffness and damping matrices K_b and C_b, respectively, the matrix differential equation can be expanded to the state space form

$$\frac{d}{dt}\begin{bmatrix} \underline{x} \\ \dot{\underline{x}} \end{bmatrix} = \left[\begin{array}{c|c} 0 & I \\ \hline -M^{-1}(K + K_b) & -M^{-1}C_b \end{array}\right]\begin{bmatrix} \underline{x} \\ \dot{\underline{x}} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}\underline{f} \tag{2}$$

Magnetic bearings may be described by a linearized force law relating the bearing force to displacement of the journal and symmetric perturbation of the bearing coil currents [8]:

$$f_a = -K_x x_a - K_i i \tag{3}$$

The perturbation current, i, is related to the motion of $node_s$ of the rotor by the linear control law

$$I = G(s)X_s \tag{4}$$

The bearing force is applied to $node_a$ while the sensor providing the feedback control senses the position of $node_s$. If $node_a$ and $node_s$ are different (as they generally are), then collocation error is introduced.

To incorporate the bearing description of equation (3) into the state space description, the frequency domain equation governing the perturbation current must be transformed back to the time domain. Thus,

$$i^{(n)} + \beta_{n-1} i^{(n-1)} + \ldots + \beta_1 i^{(1)} + \beta_0 i = \alpha_m x_s^{(m)} + \ldots + \alpha_1 x_s^{(1)} + \alpha_0 x_s \tag{5}$$

in which the superscript (i) denotes the i^{th} time derivative and the coefficients α_i and β_i are derived from the transfer function G(s):

$$G(s) = \frac{\alpha_m s^m + \ldots + \alpha_1 s + \alpha_0}{s^n + \beta_{n-1} s^{n-1} + \ldots + \beta_1 s + \beta_0} \tag{6}$$

In general, the numerator of G(s) will be of order higher than one, implying that equation (5) will involve higher derivatives of x_s than are available in the state vector. This problem is circumvented as G(s) can always be expanded, via partial fraction expansion, into a sum of transfer functions all of which are zero or first order in the numerator. In this manner, a bearing having a second or higher order transfer function numerator can be split into several separate bearings acting at the same point, each having first or zero order transfer function numerators.

So equation (5) can, without loss of generality, be written as:

$$i^{(n)} = \alpha_0 x_s + \alpha_1 x_s^{(1)} - \beta_0 i - \beta_1 i^{(1)} - \ldots - \beta_{n-1} i^{(n-1)} \qquad (7)$$

Transforming to state space form,

$$\frac{d}{dt}\begin{bmatrix} x_s \\ x_s^{(1)} \\ i \\ i^{(1)} \\ \vdots \\ i^{(n-1)} \end{bmatrix} = \left[\begin{array}{cc|cccc} 0 & 1 & & & & \\ * & * & & & & \\ \hline & & 0 & 1 & & \\ & & 0 & 0 & 1 & \\ & & & & \ddots & 1 \\ \alpha_0 & \alpha_1 & -\beta_0 & -\beta_1 & \cdots & -\beta_{n-1} \end{array}\right] \begin{bmatrix} x_s \\ x_s^{(1)} \\ i \\ i^{(1)} \\ \vdots \\ i^{(n-1)} \end{bmatrix} \qquad (8)$$

Equation (8) is incorporated into equation (1) by augmenting the state vector with the required current terms:

$$\frac{d}{dt}\begin{bmatrix} \underline{x} \\ \hline \dot{\underline{x}} \\ \hline i \\ i^{(1)} \\ \vdots \\ i^{(n-1)} \end{bmatrix} = \left[\begin{array}{c|c|cccc} 0 & I & & & & \\ \hline -M^{-1}(K + K_b) & -M^{-1}C_b & & & & \\ \hline & & 0 & 1 & & \\ & & & & 1 & \\ & & & & \ddots & 1 \\ \alpha_0 & \alpha_1 & -\beta_0 & -\beta_1 & \cdots & -\beta_{n-1} \end{array}\right] \begin{bmatrix} \underline{x} \\ \hline \dot{\underline{x}} \\ \hline i \\ i^{(1)} \\ \vdots \\ i^{(n-1)} \end{bmatrix} + \begin{bmatrix} 0 \\ \hline M^{-1} \\ \hline 0 \end{bmatrix} f \qquad (9)$$

Finally, equation (3) forms a feedback control law for the force term in equation (9):

$$f = \left[\begin{array}{c|c|cccc} 0 \cdots 0 \; -K_x \; 0 \cdots 0 & & -K_i & 0 & \cdots & 0 \end{array}\right] \begin{bmatrix} \underline{x} \\ \hline \dot{\underline{x}} \\ \hline i \\ i^{(1)} \\ \vdots \\ i^{(n-1)} \end{bmatrix} \qquad (10)$$

An example will illustrate the assembly process. The dynamics of an axisymmetric nongyroscopic three mass rotor model is described by:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \ddot{\underline{x}} + \begin{bmatrix} 2 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix} \underline{x} = \underline{f} \qquad (11)$$

There are two magnetic bearings, acting on stations 1 and 3. The bearing acting on station 1 has no collocation error, while that acting on station 3 senses station 2. They are described by:

$$f_1 = 5x_1 - 2i_1, \quad f_3 = 3x_3 - 3i_2 \qquad (12)$$

The transfer functions relating the perturbation currents to the rotor motion are

$$G_1(s) = \frac{0.1s + 2}{s^3 + 4s^2 + 5s + 3}, \quad G_2(s) = \frac{5}{s^2 + 6s + 2} \qquad (13)$$

After expanding to state space form, incorporating the bearing transfer functions, and implementing the feedback of equation (10), the system differential equation becomes

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \hline \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \hline i_1 \\ \dot{i}_1 \\ \ddot{i}_1 \\ \hline i_2 \\ \dot{i}_2 \end{bmatrix} = \left[\begin{array}{ccc|ccc|ccc|cc} & & & 1 & & & & & & & \\ & & & & 1 & & & & & & \\ & & & & & 1 & & & & & \\ \hline 1.5 & 1.5 & -.5 & & & & -1 & & & & \\ .75 & -1.25 & .5 & & & & & & & & \\ -1 & 2 & 2 & & & & & & & -3 & \\ \hline & & & & & & & 1 & & & \\ & & & & & & & & 1 & & \\ 2 & & & .1 & & & -3 & -5 & -4 & & \\ \hline & & & & & & & & & & 1 \\ & 5 & & & & & & & & -2 & -6 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \hline \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \hline i_1 \\ \dot{i}_1 \\ \ddot{i}_1 \\ \hline i_2 \\ \dot{i}_2 \end{bmatrix} \qquad (14)$$

Model of Laboratory Apparatus

For the purposes of this investigation, a ten mass single plane model was used. The coupling is modeled as a simple bearing. All rotational degrees of freedom were removed from the model using static condensation.

Each magnetic bearing has eight poles. The stators are constructed of "soft" silicon magnet iron and are unlaminated. Hysteresis losses appear, on the basis of static measurements, to be substantial and are modeled as providing an added lag time constant of 1.0 msec in the control loop. The 1.0 msec value was determined by comparing the experimentally determined stability threshold with that predicted by the mathematical model corresponding to $K_p = 2.5$ and $K_d = 2.85$ with the motor coupling disconnected. This linear model of a highly nonlinear effect appears to be adequate for small motions of the shaft, but may introduce substantial error for large motions. The bearing coefficients are estimated (including losses due to leakage and fringing) as:

$$K_x = -36000 \text{ N/m} \quad , \quad K_i = 206 \text{ N/A} \tag{15}$$

The control algorithm employed is simple proportional-derivative (PD) feedback. The actual controllers used are adjustable, having a transfer function defined by

$$G_c(s) = 0.05 \frac{\alpha_1 K_p s^2 + (\alpha_2 K_p + \alpha_3 K_d) s + K_p}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)} \tag{16}$$

The various coefficients are reported in Table 1. The transfer functions of the controllers have been carefully documented in earlier works on the basis of experimental measurement [7,11]. Position sensing is accomplished with eddy current probes which have a sensing gain of 7900 V/m and a time constant of 56 μsec. The output of the controller drives a transconductance amplifier which converts the control voltage to the required current to drive the bearing coils. The amplifier has a transconductance of 0.5 A/V and a pair of poles with time constant of 220 μsec due to a well damped tank circuit in the output section required for amplifier stability.

Combining the transfer functions of the probe, controller, and amplifier, and adding the pole which models the actuator hysteresis, the overall control transfer function is found to be

$$G(s) = 197.5 \frac{\alpha_1 K_p s^2 + (\alpha_2 K_p + \alpha_3 K_d) s + K_p}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)(\tau_4 s+1)(\tau_5 s+1)(\tau_6 s+1)(\tau_7 s+1)} \tag{17}$$

Table 1. Controller Coefficients

Symbol	Value	Symbol	Value
α_1	2.0×10^{-8} sec^2	τ_3	47 μsec
α_2	0.320 msec	τ_4	56 μsec
α_3	2.20 msec	τ_5	220 μsec
τ_1	220 μsec	τ_6	220 μsec
τ_2	100 μsec	τ_7	1.0 msec

Impulse Response Simulation

There are several methods for simulating the impulse response of a system from a state space model. Among these are numerical techniques such as Runge Kutta or Euler time integration [12], direct analytic solutions such as Lukes' ABC algorithm [13], and frequency domain techniques based on the Laplace transform [14]. In this investigation, the direct analytic approach was adopted both for its speed in obtaining simulations and for the ease with which an impulse of finite duration can be represented. The actual time history of the impulse applied by the experimental apparatus was unavailable, but a 1.0 msec impulse was modeled.

Conclusions

Figure 2 shows the measured experimental and the predicted theoretical response of the rotor at the outboard sensor subjected to an impulse applied at the outboard end (end opposite motor). These data are for controller settings, $K_p = 2.5$ and $K_d = 4.5$ corresponding to proportional and damping gains of 117 N/mm and 0.79 N-s/mm, respectively. The general forms of the two curves agree as well as the fundamental frequencies (first three criticals are 2000, 3800, and 5500, experimentally measured and 2190, 4350, and 5768 rpm, theoretically predicted).

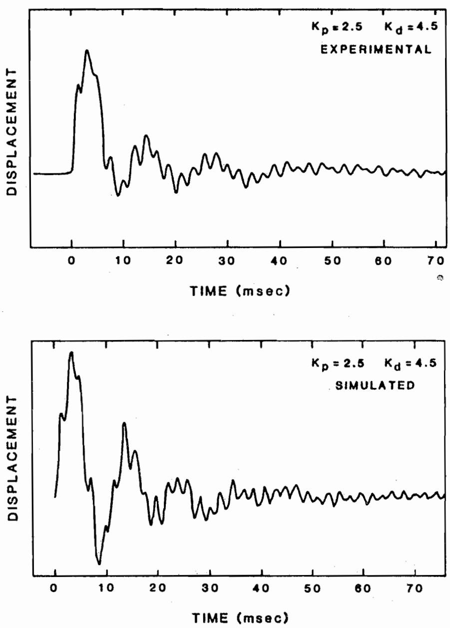

Figure 2. Experimental and Calculated Response with $K_d = 4.5$

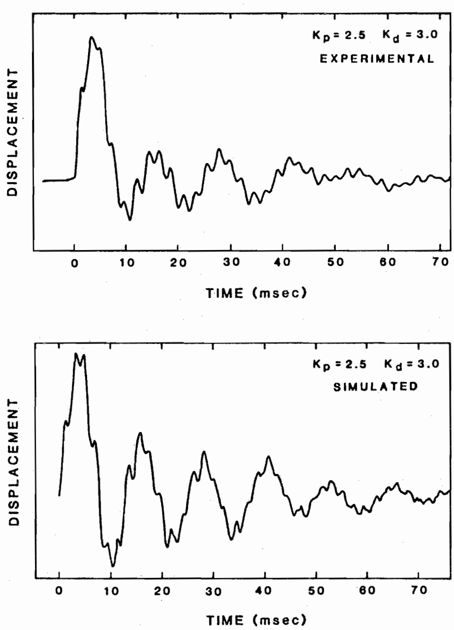

Figure 3. Experimental and Calculated Response with $K_d = 3.0$

Figure 3 shows results similar to those of Fig. 2 except with a reduced derivative gain of $K_d = 3.0$. The stability threshold for $K_p = 2.5$ is $K_d \simeq 2.4$, so the reduction from 4.5 to 3.0 would be expected to produce substantially reduced damping. This is supported by both the experimental and simulated results of figure 3 where, again, concurrance of theory and experiment is quite good.

This work illustrates that by having a good description of the control circuitry and by making a detailed model of the system, even relatively complex transient behavior can be predicted accurately.

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