Design of a Controller for Magnetic Levitation

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SUMMARY

In this work we will apply a linear quadratic regulator for the controller of the magnetic levitation of a rigid body. A successfull implementation with a diferentiator to estimate the velocity is showed, using a linear and not noisy displacement sensor. Another implementation with a pragmatic control policy that worked well with a noisy and strongly nonlinear sensor is presented.

1.0) INTRODUCTION

Robustness is a well-known property of optimal linear quadratic control (LQR). Guaranteed phase and gain margins of respectively, 60 degrees and -6 dB to infinity is a measure of robustness with this control policy [1], [2]. For a system

$$\dot{x}=Ax+Bu, \qquad x(0)=x_0, \qquad v=Cx \tag{1.1}$$

in which we attempt to minimize the functional

$$J=\int_0^{+\infty}(x'Qx + u'Ru)dt \tag{1.2}$$

with Q, R symmetric and respectively, semi-definite and positive definite matrices, the steady state optimal solution doesn't depend on the initial condition x_0 and it is of the state feedback form:

$$u(t)=Fx(t), \qquad F=-R^{-1}B'P \tag{1.3}$$

where P is the unique solution of the Riccati equation [3]. Furthermore, controllability of (A,B) ensures stability of the closed-loop system $\dot{x}=(A+BF)x$.

In this work we will apply a LQR for the controller of the magnetic levitation of a rigid body, [4], [5], for two cases. In the first one we will use a linear displacement sensor, and in the second one we will use a noisy and nonlinear sensor.

2.0) PLANT IDENTIFICATION

The state variable system representation will depend on the knowledge about the magnetic actuator, sensors, power amplifier and body weight. In this section, the plant identification and its linearization are described.

2.1) Magnetic actuator

The following law will be considered for the magnetic actuator force:

$$f=C(I/X)^2 \qquad (2.1)$$

where: f: force, (N)
I: coil current, (A)
X: displacement (m) and
C: magnetic actuator constant.

The magnetic actuator constant C can be found through experimental methods, [6], analytical methods (in some cases) or by finite element approach. We used an experimental method, in which the current I is obtained for a constant f, at each gap X. Thus, we obtained $C=2.223E-5\ Nm^2/A^2$.

2.2) Displacement sensors

We used an eddy current sensor and an inductive sensor. The first one presents a good linearity while the second one is highly nonlinear. In the test rig we got the calibration curves (fig. 1):

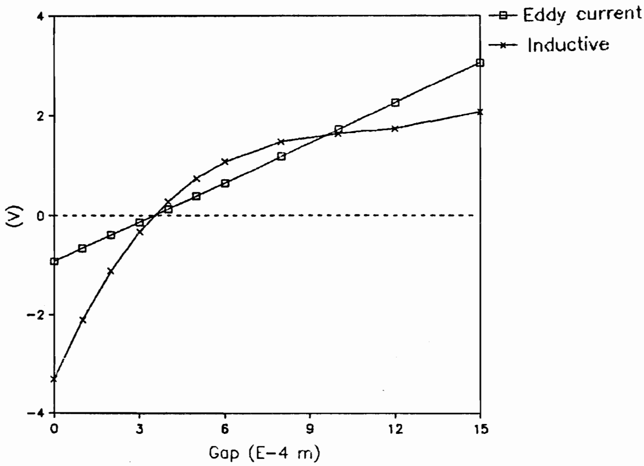

Fig. 1 - Sensors calibration curves

Eddy current sensor: $v=2625X-0.92$, (2.2.1)

Inductive sensor: $v=-3.32+1.33E4X-1.22E7X^2+3.87E9X^3$ (2.2.2)

2.3) Power amplifier

The power amplifier used is linear and has a very flat frequency response. Then, we can describe it as a constant gain $K_a=0.42$ A/V.

2.4) State variable system representation

The linearized force around levitation gap is:

$$f= f(Io,Xo) +\frac{df}{dX}\Big|_{\substack{X=Xo,\\ I=Io}}(X-Xo) \quad +\frac{df}{dI}\Big|_{\substack{X=Xo,\\ I=Io}}(I-Io) + \quad .... \tag{2.4}$$

where Xo and Io are respectively the levitation gap and levitation current. If we write x=X-Xo and i=I-Io, and use (2.1), we get:

$$f \simeq CK_k^2-K_x x+K_m i \tag{2.4.1}$$

where: $K_x=2CIo^2/Xo^3$; $K_m=2CIo/Xo^2$; $K_k=Io/Xo$.

Thus, the differencial equation for the system linearized around levitation gap is $M\ddot{x}=K_x x-K_m i$ (M: mass of body). In matricial representation (1.1):

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix} = \begin{bmatrix}0 & 1\\ K_x/M & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix}+\begin{bmatrix}0\\ -K_m/M\end{bmatrix}i \tag{2.4.2}$$

We can change the mechanical state variables by the eletrical state variables:

$$\begin{bmatrix}\dot{v}\\ \ddot{v}\end{bmatrix} = \begin{bmatrix}0 & 1\\ K_x/M & 0\end{bmatrix}\begin{bmatrix}v\\ \dot{v}\end{bmatrix}+\begin{bmatrix}0\\ -K_m K_d K_a/M\end{bmatrix}Va \tag{2.4.3}$$

with $K_d= \frac{dv}{dX}\Big|_{X=Xo}$ and Va: control voltage output.

In this way, when evaluating the state feedback to get a LQR, we have to calculate the gains for displacement and velocity signals.

3.0) CONTROLLER DESIGN: EDDY CURRENT SENSOR CASE

Choosing Xo=0.35 mm for the levitation gap we obtained the following matrices for the system:

$$A=\begin{bmatrix}0 & 1\\ 5.604E+4 & 0\end{bmatrix}, \quad B=\begin{bmatrix}0\\ -2.007E+5\end{bmatrix}, \quad C = \begin{bmatrix}1\\ 0\end{bmatrix}' . \tag{3.1}$$

It is easy to demonstrate that the system above is controlable and observable, [5].

We evaluated the LQR feedback matrices F for different choices of the ponderation matrices Q and R. Through computer simulation of the nonlinear system model, we found the best gains.

$$Q=\begin{bmatrix}1 & 0\\ 0 & 3.0E-4\end{bmatrix}, \quad R= [\ 1\ ], \quad F= [-1.32 \quad -1.77E-2] . \tag{3.2}$$

We got a good performance with this choice of F (Fig. 2, curve 1). For that ponderation the levitation was fast and without overshoot. The simulation results were very close to those ones we got in the test rig, where we used a differentiator to produce a velocity estimation. The implemented transfer function and its approximation for low frequencies were:

$$G(s)=1.32+\frac{2.7966E8s}{s^2+11.3E3s+1.58E10} \simeq 1.32+1.77E-2s \tag{3.3}$$

3.1) Set-point regulator (SPR)

In order to obtain greater bearing stiffness around the levitation gap, we tried to increase the closed-loop gains using another F, but the levitation transient became worse due to growth of the overshoot (Fig. 2, curve 2). Overshoot appears because the magnetic actuator can't produce repulsive forces to brake the body.

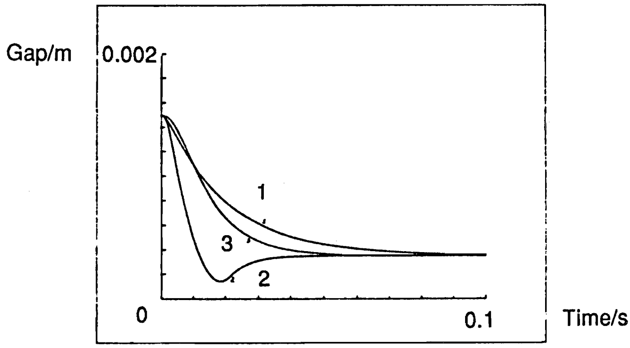

Fig. 2 - Levitation Transient: Gap(m) X Time (s)
1) F11=1.32; 2) F11=3.42 3) F11=3.42 with SPR

High closed-loop gains usually produce high currents with abrupt variation in the levitation transient. When magnetic material is submited to that flux variation, it becomes magnetized (magnetic remnant). This will produce greater overshoot in the test rig than that one obtained by computer simulation (where the magnetic remnant model is not available).

A solution found for this problem was to get slow levitation transients with high closed-loop gains. This was possible due to robustness of the LQR during transient levitation and by a slowly variable signal which was added to the set-point D.C. voltage for gap Xo. This corresponds to the following block diagram:

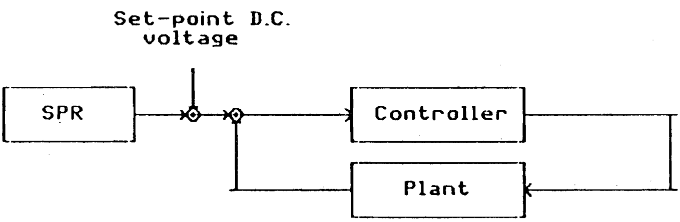

Fig. 3 - Set-point regulator (SPR) control block diagram

The SPR has been used in order to keep the control error signal very small during all the levitation transient and consequently to produce small body acceleration and coil current, even with high closed-loop gains (Fig. 2, curve 3).

4.0) CONTROLLER DESIGN: INDUCTIVE SENSOR CASE

In section 3 we presented a controller with a differentiator to produce a velocity estimation. This worked well because the Eddy current sensor is linear and not noisy. However, the same control policy presented a bad performance when using an inductive sensor. This is due to the noisy nature of that sensor. Furthermore, the inductive sensor is strongly nonlinear (Fig 1).

The controller for this case must be robust (specially due to the non-linearity of the sensor) and the closed-loop system must have good noise rejection properties.

The first possible approach is the design of an observer-based compensator, considering the closed-loop robustness. Transfer function computations will be necessary to determine the noise and mechanical disturbance rejection. For final verification, the sensor non-linearity will be taken in account in computer simulation.

Considering the differential equations of the system and a observer-controller topology [7] with a full state observer, we have:

$$\begin{bmatrix}\dot{x}\\ \dot{w}\end{bmatrix} = \begin{bmatrix}A & BF\\ -GC & A+GC+BF\end{bmatrix}\begin{bmatrix}x\\ w\end{bmatrix} + \begin{bmatrix}B\\ B\end{bmatrix}v(t) + \begin{bmatrix}B\\ 0\end{bmatrix}q(t) + \begin{bmatrix}0\\ -G\end{bmatrix}r(t),$$

$$y = \begin{bmatrix}C & 0\end{bmatrix}\begin{bmatrix}x\\ w\end{bmatrix}. \qquad (4.1)$$

where r(t) is the sensor noise, v(t) is a known input and q(t) is an unknown mechanical disturbance. We can use the similarity transformation in order to compute the transfer functions:

$$\bar{A}_c = T^{-1}A_cT =$$

$$\begin{bmatrix}0 & I\\ I & -I\end{bmatrix}\begin{bmatrix}A & BF\\ -GC & A+GC+BF\end{bmatrix}\begin{bmatrix}I & I\\ I & 0\end{bmatrix} = \begin{bmatrix}A+BF & GC\\ 0 & A+GC\end{bmatrix}. \qquad (4.2)$$

From matrix $\bar{A}_c$ (4.2) the so called deterministic separation property is clear. Indeed, the tringular-block form of (4.2) implies that the poles of $\bar{A}_c$ are the poles of (A+BF) and (A+GC). The controlability of (A,B) and the observability of (C,A) ensures arbitrary allocation of the poles of (A+GC) and (A+BF), respectively. For instance, we can use LQR algorithm to determine the optimal feedback F and use the dual system to determine a G' so that (A'+C'G') has adequate poles. Unfortunately, this ensures stability but there is no guarantee of robustness with observer-based controllers [8], [9]. In the new base defined by the similarity transformation (4.2) is very easy to compute all the transfer functions from v, q and r to x. Moreover, it's clear that the controler's transfer function is $L(s)=-F(sI-(A+BF+GC))^{-1}G$, and the plant's transfer function is $H(s)=C(sI-A)^{-1}B$. So it's not difficult to determine the open-loop L(s)H(s) (so the gain and phase margins) and noise and disturbance rejection. Then, we can proceed in the design with several choices of F and G, comparing the results of each one.

For our particular problem, we observed that the robustness and noise rejection of the closed-loop system led to relatively low gains of the controller. On the other hand we required that the controller must levitate the body from its start position, which is 1.15 mm distant from the levitation gap 0.35 mm. Due to the non-linearity of the sensor and actuator, the linearized open-loop gain around the start position is many times lower than that one around the levitation gap. In practice, the gain adjustment of the controller became almost impossible, because it must be high enough to produce a current in the magnetic actuator in order to levitate the body, but it has to be low enough because of the noise and stability restrictions. We solved this problem by adding an integrator to the system, and then proceeding with a LQR policy. The plant to be controlled (2.4.3) was a little bit different from the one in the section 3, and is given by:

$$A1=\begin{bmatrix} 0 & 1 \\ 5.6E+4 & 0 \end{bmatrix}, \quad B1=\begin{bmatrix} 0 \\ -1.3E+5 \end{bmatrix}, \quad C1=\begin{bmatrix} 1 \\ 0 \end{bmatrix}'. \tag{4.3}$$

We added an integrator in the way that a new input was defined. The complete new plant will be:

$$A=\begin{bmatrix} 0 & 1 & 0 \\ 5.6E+4 & 0 & -1.3E+5 \\ K & 0 & 0 \end{bmatrix}, \quad B=\begin{bmatrix} 0 & 0 \\ -1.3E+5 & 0 \\ 0 & -1 \end{bmatrix}. \tag{4.4}$$

This corresponds to the following block diagram:

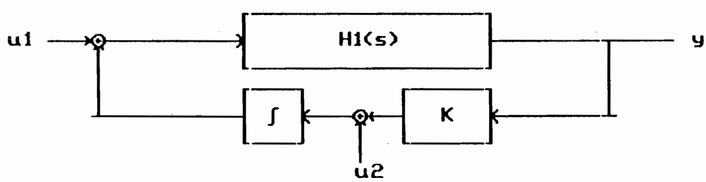

Fig. 4 - Integrator system block diagram - $H1(s)=C1(sI-A1)^{-1}B1$

We added an integrator and a gain K to the original system, defining a new input u2. The exact value of K was determined in a trial and error fashion. To this new system defined by (4.4) we evaluate the LQR feedback matrices with differents choices of Q, R and K in functional (1.2) and in the system (4.4). The main idea was to have a strong ponderation of the first input u1 compared with the second input u2. This was done in order to have lower gains in the proportional and derivative parts of the controller, but great gain in the integral part, which will produce high D.C. gain. With the choices:

$$Q=\begin{bmatrix} 400 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad R=\begin{bmatrix} 200 & 0 \\ 0 & 1 \end{bmatrix}, \quad K=50. \tag{4.5}$$

the optimal feedback matrix was found to be

$$Fo=\begin{bmatrix} 2.31 & 1.69E-2 & 0.187 \\ -2.74 & 2.88E-4 & 8.27 \end{bmatrix}. \tag{4.6}$$

The reader can verify that the matrix Fo with $Fo_{22}=0$ will produce the same closed-loop poles. The controller transfer function would be $L(s)=1.69E-2s-2.31+38.42/(s+8.27)$.
The implemented transfer function was:

$$L(s)=L_o(s)+L_i(s) \tag{4.7}$$

where: $L_o(s)=\dfrac{-2.08E8(1.69E-2s-2.31)}{s^2+5.02E4s+2.08E8}$ and $L_i(s)=\dfrac{38.42}{s+8.27}$

The $L_o(s)$ produces an estimation of the proportional and derivative part of the control. Its poles were selected by trial and error, in order to have robustness and good dynamic response, dictated by the closed-loop poles. The $L_i(s)$ is the integral part of the controller.

In order to check the closed-loop system behavior, several computer simulations were performed. We check the controller performance considering sensor and actuator non-linearities, mechanical disturbances and sensor noise. In this way the controller (4.7) was found to be a good choice. As an example of simulation, we show in the figure 5 a typical levitation curve with the set-point regulator. These results were very close to the experiment results .

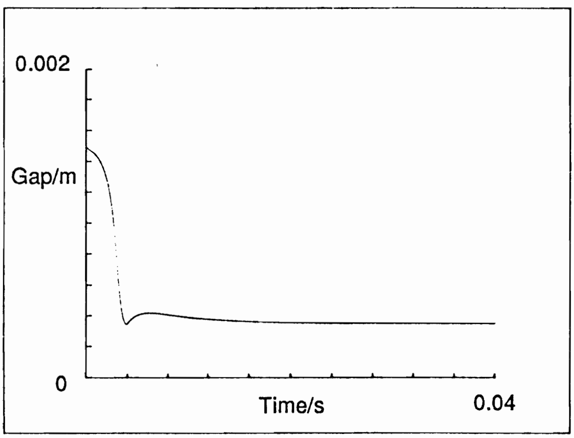

Fig. 5 - Levitation Transient: System with integrator

5.0) CONCLUSION

In section 3 we showed that, in some cases, the differentiator is a simple manner to produce a velocity estimation and an optimal control can be implemented with good closed-loop performace. In section 4, we showed that, in the case of a noisy and non-linear sensor, a simple design method was able to produce a robust controller. In this case an integrator was added and its pole was determined by an LQR algorithm. So we can call the controller of section 4 as an "optimal PID control".

We showed the great influence in controller design of the displacement sensors.

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