# Analysis on the Operating Stability of a Magnetic Bearing

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ABSTRACT

In this paper, the production mechanism of self-excited vibration of a magnetic bearing-rotor system is explained theoretically, and the related operating stability is investigated. A clan of the stable boundary curve derived theoretically is basically in agreement with the experiments. Those provide a foundation to choose the parameters of a rotor system, such as the parameters of rotor, damper and magnetic bearing. In this paper, a prototype device could be run up to a high speed(30,000r.p.m.) stably and repeatedly.This device possesses a rotor which weighs 10 kilogram and is suspended with a controlled magnetic bearing.

## I. INTRODUCTION

The rotor controlled with a magnetic bearing is easily produced self-excited vibration which interrupts the operating stability and which comes from the states of the controlling circuit and the hysteresis of the material, when it turns with a high speed. Therefore, the technical problem of how to eliminate self-excited vibration must be solved. Early in 1946, J.W.Beams[1] had possessed a specific device to avoid self-excited vibration in a ultra-centrifuge, and then other scientists also studied the similar devices. G.Schweitzer[2] studied another kind of magnetic bearing, possessing radial rigidity and damping.

The device described in this paper was developed early before, and its construction is very simple, but its mechanical performance was satisfied. A simple description of this device is shown in Fig.1. The rotor is suspended by an attraction of iron core which is immersed into damping oil and is hung at the axis of the hole coil through a steel string. Then we could have the fairly good result to avoid self-excited vibration if we change the parameters of the diameter of steel string and viscosity of the damping oil. The philosophy of choosing all these parameters will be given theoretically later.

## II. THE MECHANICAL-MODAL OF MAGNETIC BEARING-ROTOR SYSTEM

When the theoretical analysis is undertaking, a suitable mechanical-model should be built by simplify of the mechanical device. The basic hypothesis will be given as follows.

(1)Both rotor and iron core are regarded as particles, neglecting their moments of inertia.

(2)In equilibrium state, the mass centre of the electromagnetic iron core, the mass centre of the measured point down side are all in a same vertical line.

(3)Only the horizontal movements of the rotor and iron core are considered, when rotor is turning around.

(4)Being not in its equilibrium place, the rotor will be acted by a restored force which is proportional to the relative displacement between the rotor and the iron core.

(5)The damping force acting on the rotor is viscous when rotor moves in the oil. The symbol applied in this paper is given in the following:

$m_1$---The mass of the electromagnetic iron core (kg).

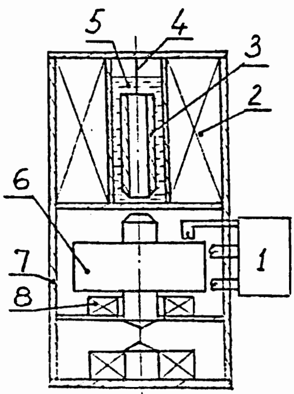
$m_2$---The mass of the rotor (kg).

$f_0$---The restored force of the rotor (N).

$k_1$---Equivalent stiffness of steel string (N/m).

$k_0$---Equivalent stiffness of magnetic force (N/m).

$d_1$---External damping coefficient from the viscosity of the oil (N/s.m).




1.vibration meter
controlling box(circuit)
speed meter
2.controlling coil
3.iron core
4.steel string
5.damping oil
6.rotor
7.vacuum casing
8.motor

Fig.1. Experimental rig

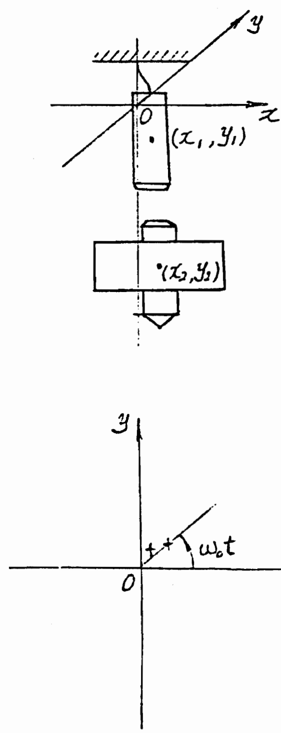



Fig.2. Coordinate system

$d_o$---Internal damping coefficient of the magnetic damping (N/s.m).

$\omega_o$---Angular speed of the rotor (rad/s).

$\bar{m}$---The dimensionless mass ($\bar{m}=m_1/m_o$).

$\bar{d}$---The dimensionless damping ($\bar{d}=d_1/d_o$).

$\bar{k}$---The dimensionless stiffness ($\bar{k}=k_1/k_o$).

## III. EQUATION OF MOTION

As shown in Fig.2, a coordinate system XOY is established fixedly on the foundation, "$x_1, y_1$" represents the coordinates of the mass centre of the electromagnetic iron core,"$x_2, y_2$" represents the ones of the rotor. By these, we obtain equation of motion of the iron core and rotor:

$$m_1\ddot{z}_1 + d_1\dot{z}_1 + k_1 z_1 - f_o = 0$$
$$m_2\ddot{z}_2 + f_o = 0 \qquad (1)$$

Where $z_1, z_2$ are the following complex expression:

$$z_1 = x_1 + iy_1 = \zeta_1 e^{i\omega_o t}$$
$$z_2 = x_2 + iy_2 = \zeta_2 e^{i\omega_o t} \qquad (2)$$

From that we conclude

$$\dot{z}_k = \dot{\zeta}_k + i\omega_o\zeta_k e^{i\omega_o t}$$
$$\dot{\zeta}_k = \dot{z}_k - i\omega_o z_k e^{i\omega_o t} \qquad k=1,2 \qquad (3)$$

Assuming the rotor and the iron core move corresponded, then obtain:

$$f_o = [k_o(\zeta_2 - \zeta_1) + d_o(\dot{\zeta}_2 - \dot{\zeta}_1)]e^{i\omega_o t}$$
$$= k_o(z_2 - z_1) + d_o(\dot{z}_2 - \dot{z}_1) - i\omega_o d_o(z_2 - z_1) \qquad (4)$$

Considering Eq.(2,4), Expression(1) becomes:

$$m_1\ddot{z}_1 + (k_1 + k_o)z_1 - k_o z_2 + (d_1 + d_o)\dot{z}_1 - d_o\dot{z}_2 - i\omega_o d_o(z_2 - z_1) = 0$$
$$m_2\ddot{z}_2 + k_2 z_2 - k_o z_1 + d_o\dot{z}_2 - d_o\dot{z}_1 - i\omega_o d_o(z_2 - z_1) = 0 \qquad (5)$$

Introducing the symbols :

$$\Omega_1^2 = k_1/m_1 \; , \quad \Omega_o^2 = k_o/m_1 \; , \quad \Omega_2^2 = k_o/m_2$$
$$\varepsilon n_1 = d_1/m_1 \; , \quad \varepsilon n_o = d_o/m_1 \; , \quad \varepsilon n_2 = d_o/m_2 \qquad (6)$$

Assuming that the damping is light, the quantity $\varepsilon$ is a small one, the Eqs.(5) are transformed as

$$\ddot{z}_1 + (\Omega_1^2 + \Omega_o^2)z_1 - \Omega_o^2 z_2 = \varepsilon[n_o\dot{z}_2 - (n_1 + n_o)\dot{z}_1 + i\omega_o n_o(z_1 - z_2)]$$
$$\ddot{z}_2 + \Omega_2 z_2^2 - \Omega_2 z_1^2 = \varepsilon[n_o\dot{z}_1 - n_2\dot{z}_2 + i\omega_o n_2(z_2 - z_1)]$$

and the related homogeneous form is

$$\ddot{z}_1 + (\Omega_1^2 + \Omega_0^2)z_1 - \Omega_0^2 z_2 = 0$$
$$\ddot{z}_2 + \Omega_2^2 z_2 - \Omega_2^2 z_1 = 0 \qquad (8)$$

Assuming the solution of Eqs.(8) possesses a form

$$z_1 = z_{1k} e^{i\omega t}$$
$$z_2 = z_{2k} e^{i\omega t} \qquad (9)$$

Substituting Eqs.(9) into Eqs.(8), then obtain

$$(-\omega^2 + \Omega_1^2 + \Omega_0^2)z_{1k} - \Omega_0^2 z_{2k} = 0$$
$$(-\omega^2 + \Omega_2^2)z_{2k} - \Omega_2^2 z_{1k} = 0 \qquad (10)$$

If the solution for Eqs.(9) is nontrivial, then an eigen-value equation must be satisfied

$$\begin{vmatrix} -\omega^2 + \Omega_1^2 + \Omega_0^2 & -\Omega_0^2 \\ -\Omega^2 & -\omega^2 + \Omega^2 \end{vmatrix} = 0$$

Then eigen-values are

$$\omega_k^2 = [(\Omega_1^2 + \Omega_0^2 + \Omega_2^2) \mp \sqrt{(\Omega_1^2 + \Omega_0^2 - \Omega_2^2) + 4\Omega_0^2\Omega_2^2}\ ]/2 \quad , \ k=1,2 \qquad (11)$$

Noticing Eqs.(6), these expression can be simplified

$$(\omega_k/\Omega_2)^2 = [(1 + \bar{k} + \bar{m}) \mp \sqrt{(1 + \bar{k} - \bar{m})^2 + 4\bar{m}}\ ]/(2\bar{m}) \qquad (12)$$

Substituting Eq.(11) into first one of Eqs.(10), then obtain

$$\bar{z}_k = (z_{2k}/z_{1k})\big|_{\omega_k} = [(1 + \bar{k} - \bar{m}) \pm \sqrt{(1 + \bar{k} - \bar{m})^2 + 4\bar{m}}\ ]/2$$
$$(\omega_k/\Omega_2)^2 = (1 + \bar{k} - \bar{z}_k)/\bar{m} \qquad k=1,2 \qquad (13)$$

Rewriting these eigen-solutions into matrix

$$V = \begin{bmatrix} 1 & 1 \\ \bar{z}_1 & \bar{z}_2 \end{bmatrix} \qquad (14)$$

Introducing a variable transform

$$\begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = V \begin{Bmatrix} z_{m1} \\ z_{m2} \end{Bmatrix}$$

and multiplying Eqs.(7) with $V^H$ which is a conjugate transpose matrix of V, then obtain

$$z_{m1} + \omega_1^2 z_{m1} = (\bar{z}_2 f_1(\varepsilon) - f_2(\varepsilon))/(\bar{z}_2 - \bar{z}_1)$$
$$z_{m2} + \omega_2^2 z_{m2} = (\bar{z}_1 f_1(\varepsilon) - f_2(\varepsilon))/(\bar{z}_1 - \bar{z}_2) \qquad (15)$$

where $f_1(\varepsilon)$, $_2f(\varepsilon)$ represent the right of Eqs.(5). These equations are equations of motion in standard form.

## IV. BUILDING THE STABILITY CRITERION BY THE METHOD[3,4] OF PERTURBATION

For quantity $\varepsilon$ is small, we can use perturbation method to extract the stable boundary curve obtained in Eq.(15). Eq.(15) is a linear differential equation with constant coefficient, so its solution can be expressed as follows:

$$z_{m1} = \left(\sum_0^\infty A_{1k}\varepsilon^k\right)\exp\left(i\omega_1 + \sum_1^\infty \delta_{1k}\varepsilon^k\right) + \left(\sum_0^\infty B_{1k}\varepsilon^k\right)\exp\left(i\omega_2 + \sum_1^\infty \delta_{2k}\varepsilon^k\right)$$

$$z_{m2} = \left(\sum_0^\infty A_{2k}\varepsilon^k\right)\exp\left(i\omega_1 + \sum_1^\infty \delta_{1k}\varepsilon^k\right) + \left(\sum_0^\infty B_{2k}\varepsilon^k\right)\exp\left(i\omega_2 + \sum_1^\infty \delta_{2k}\varepsilon^k\right) \quad (16)$$

Substituting Eq.(16) into Eq.(15), then taking the zero order term $\varepsilon^0$, we can obtain a solution shown in Eqs.(11),(13). If taking the first order term $\varepsilon^1$, and equalizing the corresponding coefficients, we obtain the quantities $\delta_{11}$, $\delta_{21}$, for simplification, the details of derivation are omitted. The $\delta_{11}$, $\delta_{21}$ are called modal damping which show the stability of a system, The system are stable, if $\delta_{11}<0$, $\delta_{21}<0$. We put them in following expression

$$\omega_0/\Omega_2 \le [1 + \bar{d}/(1-\bar{z}_k)^2]\sqrt{(1+\bar{k}-\bar{z}_k)/\bar{m}} \quad (17)$$

choosing the sign of equality in Eqs.(17), we obtain the stable boundary curves shown in Fig.3. The curve I is determined by k=1, corresponding the lower eigen-frequency, and the curve II is determined by k=2, cresponding the higher one. They are all expressed in non-dimensional forms. A clan of curve shown in Fig.3 indicates the influence on the stable area when the mass ratio $\bar{m}$ changes. The curves shown in Fig.4 expresses the influence of the damping ratio $\bar{d}$ to the stable area.

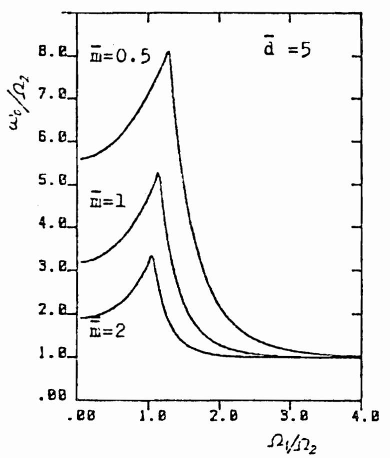



Fig.3 Stable speed $\omega_0$ increase when $\bar{m}$ decrease.

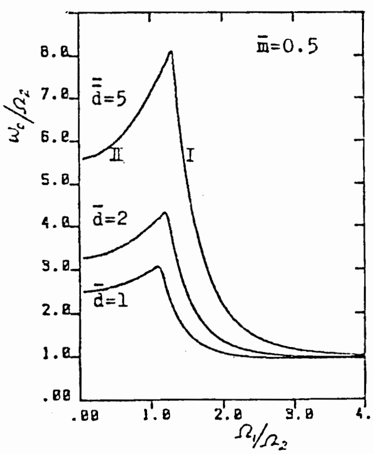



Fig.4 Stable speed $\omega_0$ increase when $\bar{d}$ increase.

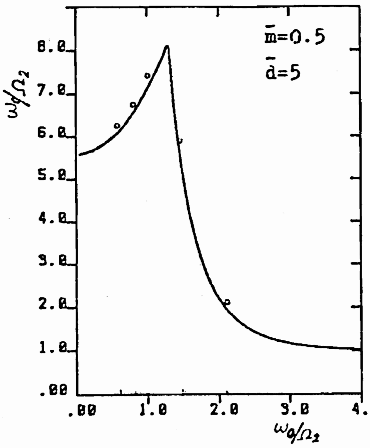



Fig.5 Comparison between theoretical curve and experimental points(∘).

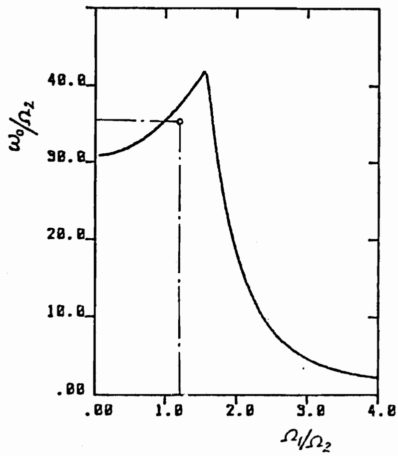



Fig.6 The work state of prototype(point ∘ )

## V. BUILDING THE STABILITY CRITERION BY THE METHOD OF ENERGY EQUILIBRIUM

In the paragraph IV, the perturbation method is used for searching the stable boundary curves. More accuracy results can be derived, if the higher orders of small quantity $\varepsilon$ are put in the calculation. Of course, much more tedious calculation must be dealt with. However, another method of energy equilibrium is simpler for this purpose[2]. We consider that in eigen-state k=1,2 the damping force from the oil is shown as

$$F_{1k} = d_1 \dot{z}_{1k} = i\omega_k d_1 z_{1k}, \qquad k=1,2$$

When the rotor and the core vibrate, the force dissipates the energy in oil, the corresponding power is indicated by

$$P_{1k} = \dot{z}^H_{1k} F_{1k} = d_1 \omega_k^2 z_{1k}^2 \qquad k=1,2 \tag{18}$$

The internal damping force from hysteresis is shown as

$$F_{2k} = d_0(\dot{z}_{2k} - \dot{z}_{1k}) - i\omega_0 d_0(z_{2k} - z_{1k}),$$

and the force introduces the energy into the system from motor, the corresponding power is as

$$P_{2k} = (\dot{z}_{2k} - \dot{z}_{1k})^H F_{2k} = d_0\omega_k(\omega_k - \omega_0)(z_{2k} - z_{1k})^2 \tag{19}$$

If the system is stable, the dissipated energy by $d_1$ must be larger or equal the energy introduced by $d_0$, i.e.

$$P_{1k} \geq P_{2k}, \qquad k=1,2 \tag{20}$$

Putting Eqs.(18),(19) into Eqs.(20), a criterion of energy for stable boundary can be transformed as

$$\omega_0/\omega_k \leq 1 + \bar{d}/(1-\bar{z}_k)^2 \qquad k=1,2 \tag{21}$$

$$\bar{z}_k = z_{1k}/z_{2k} \qquad k=1,2$$

Considering Eqs.(13 rewrite Eqs.(21) as

$$\omega_o/\Omega_2 \le [1 + \bar{d}/(1 - \bar{z}_k)^2]\sqrt{(1 + \bar{k} - \bar{z}_k)/\bar{m}} \qquad k=1,2$$

which has a same form as Eqs.(17). Therefore, the same results can be obtained from both methods, perturbation method and energy method.

## VI. OPERATION EXPERIMENT

With the axial position controlled by controlling box, and driven by the high frequency motor, the rotor is run inside the vacuum-protect casing. Using the vibration meter to check the radial amplitude and using the speed meter to cheek the rational speed of the rotor.

The rotor is made from aluminum alloy CL4 with high strength, and the attraction top of which is made from high quality low-carbon steel, the iron core from pure iron. With the safety in mind, the experiment to verify the stable boundary curve is carried on the parameters of the low-speed states ( choosing $\bar{d}=5$), the parameters of the experimental device is given in Tab.2.I. The eigen-frequency $\omega_1$ of the electromagnetic iron core is changed by changing the diameter of the steel string. The values of $\omega_1$ with respect to the steel string with length 100mm and varies diameters are given in Tab.1. Measuring five actual experimental points, on which the values is fairly approximate to the theoretical ones (just a little larger, ref. Fig.5), it is sufficient to verify that the preceding theoretical analysis is accuracy enough.

The actual prototypes requires to run in a high speed, namely a maximum rotational speed, 30,000r.p.m. is asked, (which is the limited speed of the material CL4 $\phi$240mm), i.e.,the maximum rational speed ratio $\omega_o/\Omega_2$ is demanded to reach 35.7 nearly. To reach such a high rotational speed, the achievements in the above theoretical analysis should be used to choose the suitable parameters of a rotor system, such as the parameters of rotor, damper and iron core. The iron corn is designed to be hollow (Fig.1) so as to decrease the mass ratio $\bar{m}$ and to increase the damping ratio $\bar{d}$. The parameters of the prototype is given in Tab.2.II.

The work states of the prototype is shown in Fig.6; the rotor was run up to a high speed of 30,000rpm stably and repeatedly,

Tab.1. $\Omega_2$ and the correspongding diameter of steel string

| Diameter(mm) | 1.5 | 2.0 | 2.5 | 3.5 | 5.0 |
|---|---|---|---|---|---|
| $\Omega_1$ (rad/s) | 8.8 | 11.7 | 14.5 | 20.5 | 29.0 |
| $\Omega_2$ (rad/s) | 0.6 | 0.8 | 1.0 | 1.5 | 2.1 |

Tab.2. The parameters of experimental rig

| Items | | Mass(kg) | Material | Dimension | Freq.(rad/s) | Note |
|---|---|---|---|---|---|---|
| Rotor | | 10.0 | CL4 | $\phi$240x82 | $\Omega_2=14.0$ | |
| I | Iron core | 5.0 | pure iron | $\phi$60x230 | $\Omega_1$(Tab.1) | $\bar{d}=5.0$ |
| II | Iron core | 2.5 | pure iron | $\phi$50-25x220 | $\Omega_1=17.0$ | $\bar{d}=20.$ |

nothing is found unusual, and the rotor which weighs 10 kilogram is suspended by a controlled magnetic bearing.

## VII .CONCLUSION

(1) A damping device must be used to eliminate the self-excited vibration when the magnetic bearing-rotor system turns with a much higher speed than the eigen-frequency which is about 15 Hz (900 r.p.m).

From Fig.4, we know that if the external damping $\bar{d}$ is very light, only when the ratio $\omega_o/\omega_k$ is a bit larger than 1,the system do not produce self-excited vibration. Namely, the rotor is stable only when the speed is about 15Hz (i.e. 900 r.p.m.),but up to a little higher speed,(for example,more than 900 r.p.m.),it will lost its stability.

(2) It is convenient to expand the stable area that the mass ratio $\bar{m}$ of the rotor to the iron core is decreased (Fig.3). But to decrease the ratio $\bar{m}$ is limited by the magnetism saturation of the iron.

(3) It is also very convenient to extend the stable area to increase the damping ratio $\bar{d}$. We should try to increase the external damping $d_t$. If $d_t$ is large enough, the rotor can be run up to a high speed.

(4) When the two eigen-frequencies of the rotoı and the iron core $\Omega_1$, $\Omega_2$ are matched each other, the best damping effect could be obtained.

## REFERENCE


1. Beams, J. W.; Young, J. L; Moore, J. W., The Production of High Centrifugal Fields, J. Appl. Phys., 17(1946), p886.
2. Schweitzer, G.; Stabilization of self-excited Vibration by Active Damper, Dynamics of Rotors, IUTAM Symposium, Lungby, Denmark, 1974, Springer-Verlag, Berlin, 1975, p472.
3. Tondl, A, Some Problems of Rotor Dynamics. London, Chapman and Hall, 1965.
4 Nayfeh, A. H., Introduction to Pertubation Techniques. New York, Wiley-Interscience, 1981.