Interconnected Four Poles Magnetic Bearing

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Abstract

This article analyses the adaptation of the split-wind bearingless motors structure for magnetic bearings. Preliminary theoretical results predict that a greater equivalent stiffness could be achieved when compared to the values associated with the active magnetic bearing conception traditionally described in the literature.

1 Introduction

The traditional structure of active magnetic bearings used with rotating machines, here called Type A, has four independent coils around four pair of poles resulting four independent magnetic loops as shown in Figure 1.

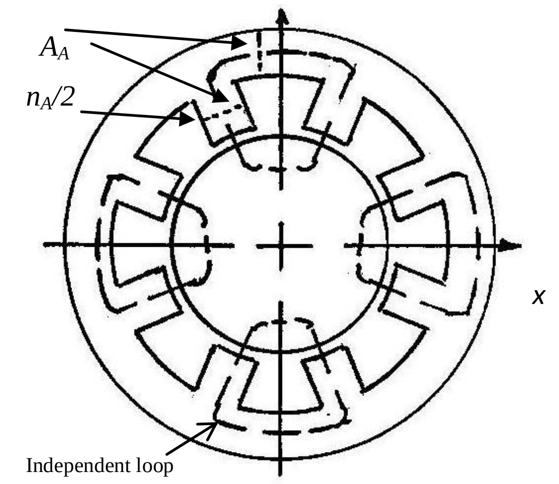

Figure 1) Type A magnetic bearing.

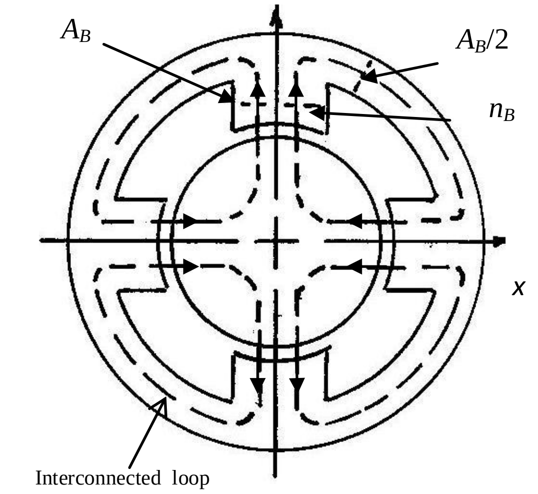

Figure 2) Type B magnetic bearing.

After a linearization procedure (Schweitzer, Bleuler, Traxler, 1994), the uncoupled magnetic bearings forces f_{Ax} and f_{Ay} are determined by the equations (1). They are derived for small displacements x and y when compared with the air gap; magnetic permeability μ_0; number of windings n_A; cross section area in the static ferromagnetic material A_A, as shown in Figure 1; bias current I_{0A}; control currents I_{xA} and I_{yA} and the length of the air gap h.

$$f_{Ax} = k_{Ap}\, x + k_{Ai}\, I_{xA} \text{ and } f_{Ay} = k_{Ap}\, y + k_{Ai}\, I_{yA}, \text{ where: } k_{Ap} = (\mu_0\, n_A^2 A_A I_{0A}^2\,)/h^3 \text{ and}$$
$$k_{Ai} = (\,\mu_0\, n_A^2 A_A I_{0A})/h^2. \tag{1}$$

An alternative structure of magnetic bearing is possible, here named Type B, also with four windings but now with interconnected magnetic loops, as depicted in Figure 2. This structure can be found in the split-windings bearingless motors researched in Brazil (Salazar, 1994), (Santisteban, 1999), (David, 2000), (Santisteban, David, Stephan, 2001). In that approach, alternate currents supply the windings, but, in the present case, continuous currents are considered. Analogously to Type A, a linearization procedure presented in the following section determine also uncoupled radial forces in equations (2).

$$f_{Bx} = k_{Bp}\, x + k_{Bi}\, I_{xB} \text{ and } f_{By} = k_{Bp.}\, y + k_{Bi}\, I_{yB}, \text{ where: } k_{Bp} = (2\mu_0 n_B^2 A_B I_{0B}^2)/h^3 \text{ and}$$

$k_{Bi} = (2\mu_0 n_B^2 A_B I_{0B})/h^2$. (2)

Other researchers presented work with the Type B magnetic bearing concept (Kjolhede, Santos, 2007). Nevertheless, up to the present time, the authors of this article did not identify the association of Type B interconnected magnetic bearings with the uncoupled equations in (2) above. The confirmation of these theoretical results with proper experimental work would imply that the Type B magnetic bearing has significant advantages when compared to Type A. Some discussions are in section 3.

2 Equations associated to the Type B magnetic bearings.

The x and y components of a radial displacement between rotor and stator change the gap h as shown in Figure 3.

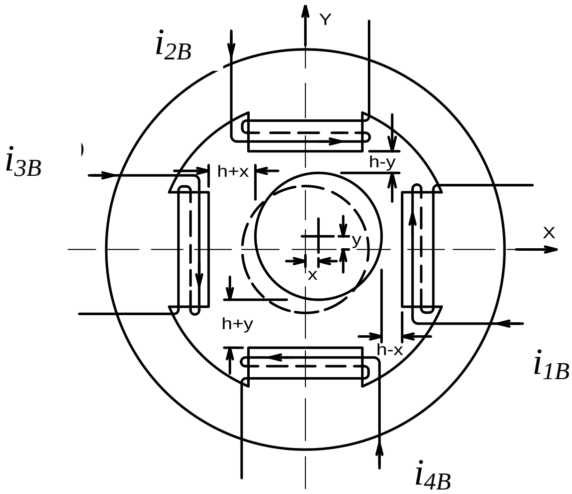

Figure 3 – Displacements and electric currents in Type B magnetic bearing.

The conventional procedure used to compensate the x and y displacement components is the application of the following differential (Schweitzer, Bleuler, Traxler, 1994) electric currents to each one of the four windings associated to the poles of the Type B magnetic bearing. I_{0B} is the bias current. I_{xB} and I_{yB} are the control currents respectively for the x and y directions.

$$i_{1B}=I_{0B}+I_{xB}\,;\;\; i_{3B}=I_{0B}-I_{xB}\,;\;\; i_{2B}=I_{0B}+I_{xB}\,;\;\; i_{4B}=I_{0B}-I_{xB} \quad (3)$$

Dotted lines in Figure 2 represent the associated magnetic flux to these currents. Mechanical forces f_{Bx} and f_{By} arise at Type B magnetic bearing. They can be determined as function of the four total magnetic flux intensities that overcome the four air gaps with cross section A_B (Figure 2): ϕ_{T1}, ϕ_{T2}, ϕ_{T3} and ϕ_{T4}.

$$f_{Bx} = \frac{1}{2\mu_0 A_B}(\phi_{T1}^2 - \phi_{T3}^2) \text{ and } f_{By} = \frac{1}{2\mu_0 A_B}(\phi_{T2}^2 - \phi_{T4}^2) \quad (4)$$

The resultant magnetic flux ϕ_{T1} in pole 1 is dependent on the magnetic fluxes ϕ_{11}, ϕ_{12}, ϕ_{13} and ϕ_{14} associated to the electric currents in each one of the four poles of the Type B interconnected magnetic bearing. The following sign convention is considered in this article: Positive fluxes flow into the rotating peace and negative fluxes flow out:

$$\phi_{T1} = \phi_{11} + \phi_{12} - \phi_{13} + \phi_{14} \quad (5)$$

Considering equivalent definitions, the resultant magnetic fluxes in the three other three poles are:

$$\phi_{T2} = -\phi_{21} - \phi_{22} - \phi_{23} + \phi_{24} \quad (6)$$

$$\phi_{T3} = -\phi_{31} + \phi_{32} + \phi_{33} + \phi_{34} \quad (7)$$

$$\phi_{T4} = -\phi_{14} + \phi_{42} - \phi_{43} - \phi_{44} \quad (8)$$

The determination of the sixteen values of ϕ_{ij} follows, where the first sub index i refers the position of each mechanical pole and the second one j to the winding associated to each pole:

The magnetic flux associated only to electric current i_{1B} defined in equation (3) is represented in Figure 4.

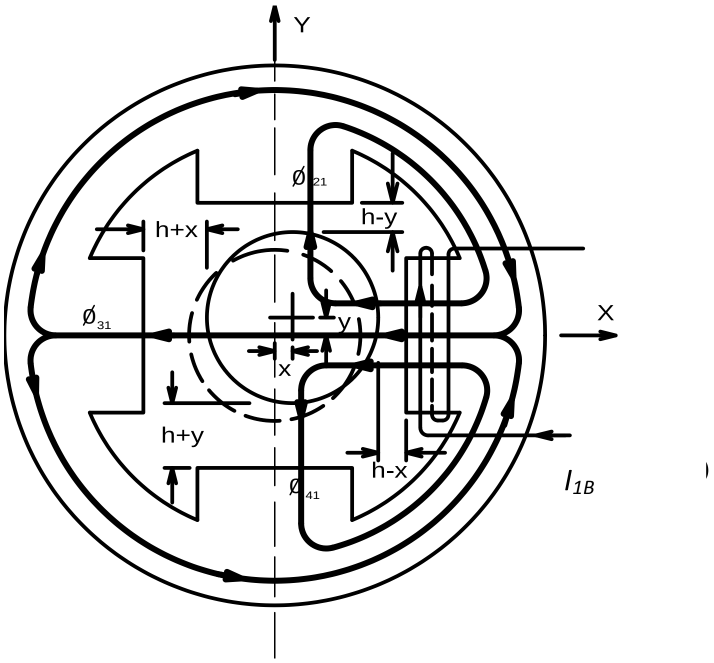

Figure 4 – Magnetic flux associated to electric currents in winding 1 of Type B magnetic bearing.

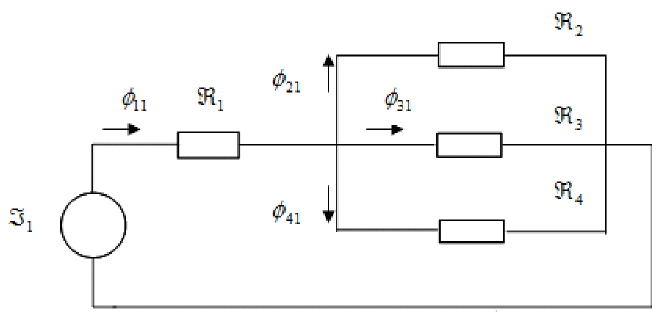

Figure 5 – Magnetic flux diagram associated to current only in winding 1 of Type B magnetic bearing.

The diagram in Figure 5 represents the magnetic circuit in Figure 4. $\Im_1$ is the magneto-motive force associated to the electric current I_{1B}. $\Re_1$, $\Re_2$, $\Re_3$ and $\Re_4$ are the reluctances associated to air gaps in each one of the four mechanical poles of the Type B magnetic bearing. These physical variables are determined by the following equations, where A_B is the cross section of the poles of Type B magnetic bearing (Figure 2).

$$\Im_1 = n_B i_{1B} \tag{9}$$

$$\Re_1 = \frac{h-x}{\mu_0 A_B}\ ;\ \Re_2 = \frac{h-y}{\mu_0 A_B}\ ;\ \Re_3 = \frac{h+x}{\mu_0 A_B}\ \text{and}\ \Re_4 = \frac{h+y}{\mu_0 A_B} \tag{10}$$

The equivalent reluctance $\Re_{eq1}$ can be then determined:

$$\Re_{eq1} = \Re_1 + \frac{1}{\frac{1}{\Re_2}+\frac{1}{\Re_3}+\frac{1}{\Re_4}} = \frac{\Re_1\Re_2\Re_3 + \Re_1\Re_2\Re_4 + \Re_1\Re_3\Re_4 + \Re_2\Re_3\Re_4}{\Re_2\Re_3 + \Re_2\Re_4 + \Re_3\Re_4} \tag{11}$$

The following auxiliary variables are defined:

$$N = \Re_1\Re_2\Re_3 + \Re_1\Re_2\Re_4 + \Re_1\Re_3\Re_4 + \Re_2\Re_3\Re_4 \tag{12}$$

$$D_1 = \Re_2\Re_3 + \Re_2\Re_4 + \Re_3\Re_4 \tag{13}$$

$$D_2 = \Re_1\Re_3 + \Re_1\Re_4 + \Re_3\Re_4 \tag{14}$$

$$D_3 = \Re_1\Re_2 + \Re_1\Re_4 + \Re_2\Re_4 \tag{15}$$

$$D_4 = \Re_1\Re_2 + \Re_1\Re_3 + \Re_2\Re_3 \tag{16}$$

With the proper algebraic operations, the following expressions are obtained for the magnetic fluxes associated to the electric current i_{1B} (3) uniquely flowing at the winding in pole 1:

$$\phi_{11} = \frac{\Im_1}{\Re_{eq1}} = n_B(I_{0B} + I_{xB})\frac{D_1}{N} \tag{17}$$

$$\phi_{21} = \phi_{11}\frac{\frac{\Re_3\Re_4}{\Re_3+\Re_4}}{\Re_2 + \frac{\Re_3\Re_4}{\Re_3+\Re_4}} = n_B(I_{0B} + I_{xB})\frac{\Re_3\Re_4}{N} \tag{18}$$

$$\phi_{31} = n_B(I_{0B} + I_{xB})\frac{\Re_2\Re_4}{N} \tag{19}$$

$$\phi_{41} = n_B(I_{0B} + I_{xB})\frac{\Re_2\Re_3}{N} \tag{20}$$

Repeating the above procedure for the electric current i_{2B} (3) at the windings in pole 2:

$$\phi_{12} = n_B(I_{0B} + I_{yB})\frac{\Re_3\Re_4}{N} \tag{21}$$

$$\phi_{22} = n_B(I_{0B} + I_{yB})\frac{D_2}{N} \tag{22}$$

$$\phi_{32} = n_B(I_{0B} + I_{yB})\frac{\Re_1\Re_4}{N} \tag{23}$$

$$\phi_{42} = n_B(I_{0B} + I_{yB})\frac{\Re_1\Re_3}{N} \tag{24}$$

Repeating the above procedure for the electric current i_{3B} (3) at the windings in pole 3:

$$\phi_{13} = n_B(I_{0B} - I_{xB})\frac{\Re_2\Re_4}{N} \tag{25}$$

$$\phi_{23} = n_B(I_{0B} - I_{xB})\frac{\Re_1\Re_4}{N} \tag{26}$$

$$\phi_{33} = n_B(I_{0B} - I_{xB})\frac{D_3}{N} \tag{27}$$

$$\phi_{43} = n_B(I_{0B} - I_{xB})\frac{\Re_1\Re_2}{N} \tag{28}$$

Repeating the above procedure for the electric current i_{4B} (3) at the windings in pole 4:

$$\phi_{14} = n_B(I_{0B} - I_{yB})\frac{\Re_2\Re_3}{N} \tag{29}$$

$$\phi_{24} = n_B(I_{0B} - I_{yB})\frac{\Re_1\Re_3}{N} \tag{30}$$

$$\phi_{34} = n_B(I_{0B} - I_{yB})\frac{\Re_1\Re_2}{N} \tag{31}$$

$$\phi_{44} = n_B(I_{0B} - I_{yB})\frac{D_4}{N} \tag{32}$$

Substituting (17) to (32) in (5) to (8) and in sequence this result in (4):

$$f_{Bx} = \frac{1}{2}\mu_0 A_B n_B^2 . q_x(h, x, y, I_{0B}, I_{xB}, I_{yB}) \tag{33}$$

$$f_{By} = \frac{1}{2}\mu_0 A_B n_B^2 . q_y(h, x, y, I_{0B}, I_{xB}, I_{yB}) \tag{34}$$

The expressions for $q_x(h, x, y, I_{0B}, I_{xB}, I_{yB})$ and $q_y(h, x, y, I_{0B}, I_{xB}, I_{yB})$ follow:

$$q_x(h, I_{0A}, x, y, I_x, I_y) = \tag{35}$$

$$= \left\{ \begin{array}{l} \left[\begin{array}{l} (I_{0B}+I_{xB})((h-y)(h+x)+(h-y)(h+y)+(h+x)(h+y))+ \\ +(I_{0B}+I_{yB})(h+x)(h+y)-(I_{0B}-I_{xB})(h-y)(h+y)+(I_{0B}-I_{yB})(h-y)(h+x) \end{array} \right]^2 - \\ -\left[\begin{array}{l} (I_{0B}-I_{xB})((h-x)(h-y)+(h-x)(h+y)+(h-y)(h+y))- \\ -(I_{0B}+I_{xB})(h-y)(h+y)+(I_{0B}+I_{yB})(h-x)(h+y)+(I_{0B}-I_{yB})(h-x)(h-y) \end{array} \right]^2 \end{array} \right\} \bullet$$

$$\bullet \left(\frac{1}{(h-x)(h-y)(h+x)+(h-x)(h-y)(h+y)+(h-x)(h+x)(h+y)+(h-y)(h+x)(h+y)}\right)^2$$

$$q_y(h, I_{0A,} x, y, I_x, I_y) = \tag{36}$$

$$= \left\{ \begin{array}{l} \left[\begin{array}{l} -(I_{0B}+I_{yB})((h-y)(h+x)+(h-y)(h+y)+(h+x)(h+y))- \\ -(I_{0B}+i_{xB})(h-y)(h+y)-(I_{0B}-I_{xB})(h+x)(h+y)+(I_{0B}-I_{yB})(h-x)(h+x) \end{array} \right]^2 - \\ -\left[\begin{array}{l} -(I_{0B}-I_{yB})((h-x)(h-y)+(h-x)(h+y)+(h-y)(h+y))+ \\ +(I_{0B}+I_{xB})(h-y)(h+y)+(I_{0B}+I_{xB})(h-x)(h+x)+(I_{0B}-I_{xB})(h-x)(h-y) \end{array} \right]^2 \end{array} \right\} \bullet$$

$$\bullet \left(\frac{1}{(h-x)(h-y)(h+x)+(h-x)(h-y)(h+y)+(h-x)(h+x)(h+y)+(h-y)(h+x)(h+y)}\right)^2$$

The function $q_x(h, x, y, I_{0B}, I_{xB}, I_{yB})$ has the following partial derivatives in the vicinity of the equilibrium point:

$$\frac{\partial q_x}{\partial x}\bigg|_{y, I_{xB}, I_{yB}=0} = -16\frac{I_{0B}^2 h(2h^2+3x^2)}{(-2h^2+x^2)^3} \text{; if } x << h, \ \frac{\partial f_x}{\partial x} = 4\frac{I_{0B}^2}{h^3}. \tag{37}$$

In any case:

$$\frac{\partial q_x}{\partial y}\bigg|_{x, I_{xB}, I_{yB}=0} = 0, \tag{38}$$

$$\frac{\partial q_x}{\partial I_x}\bigg|_{x, y, I_{yB}=0} = 4\frac{I_{0B}}{h^2}, \tag{39}$$

$$\frac{\partial q_x}{\partial I_y}\bigg|_{x, y, I_{xB}=0} = 0. \tag{40}$$

With the results from (37) to (40), the equation in (35) can be approximated near the equilibrium point:

$$q_x(h, x, y, I_{0B}, I_{xB}, I_{yB}) \cong 4\frac{I_{0B}^2}{h^3}x + 4\frac{I_{0B}}{h^2}I_x \tag{41}$$

The substitution of (41) in (33) demonstrates the first of the two equations in (2), associated to the x direction. An equivalent procedure demonstrates the equation in the y direction also in (2).

3 Conclusions

Assuming the same outside diameter of the stator, the following advantages can be identified for the Type B active magnetic bearing when it is compared to Type A:

a) The factor two present in the expressions for k_{Bp} and k_{Bi} in equation (2),

b) The cross section area A_B in equation (2) is approximately the double of the area A_A in equation (1),

c) The number of windings n_B can be increased respect to n_A.

Thus, using the same closed loop control system, the equivalent stiffness (static or dynamical) of Type B Magnetic Bearing results higher than the traditional Type A.

References

Schweitzer, G.; Bleuler, H.; Traxler, A. (1994). *Active Magnetic Bearings.* Hochschulverlag AG an der ETH Zuerich.

Salazar, A.O. (1994). *Uma proposta de Motor Elétrico sem Mancal Mecânico*. Tese de Doutorado, COPPE-UFRJ.

Santisteban, J.A. (1999). *Estudo de Influência de Carga Torsional Sobre o Posicionamento Radial de um Motor-mancal.* Tese de Doutorado, COPPE-UFRJ.

David, D.F.B. (2000). *Levitação de Rotor por Mancais-motores Magnéticos e Mancal Axial Supercondutor Auto Estável.* Tese de Doutorado, COPPE-UFRJ.

Santisteban, J.A.; David, D.F.B.; Stephan, R.M. (2001). *Active Magnetic Bearing and Induction Bearingless Machine - A Comparison*. Proceedings of The 6th Brazilian Power Electronics Conference, Vol. 2.

Kjolhede, K.; Santos, I. F. (2007). *Experimental Contribution to High-Precision Characterization of Magnetic Forces in Active Magnetic Bearings.* Journal of Engineering for Gas Turbine and Power, Vol.129. ASME.