A Stepped Magnetic Suspension System (SMSS)

Evgeni Frishman

Department of Electrical Engineering
Jerusalem College of Technology, Israel
evgeni@jct.ac.il

Abstract **—** In this article, the vertical and horizontal forces of interaction of permanent magnets in a magnetic support system (magnetic suspension, MS) are considered. Permanent magnets have a stepped structure and uniform magnetization $\vec{M}$ throughout their entire volume ($\vec{M} = \boldsymbol{const}$). The magnetic support system contains multi-row magnetic strips. The results of the comparison of the vertical and lateral forces for the classic horizontal magnetic system (HMS) are presented too. A stability factor index, the ratio of vertical to lateral force of interaction $\gamma = f_Z / f_Y$, and an effectiveness factor $\mu_{eff} = f_Z/(mg)$ are defined (where mg is the weight of the magnets per unit length). A prototype of the proposed magnetic support system was built, and measurements were performed. Analysis of the obtained data indicates that the investigated magnetic suspension system performs better than the classical horizontal MS system.

Index Terms **—** Permanent magnets, stability, stepped suspension, suspension effectiveness factor, vertical and horizontal forces.

I. INTRODUCTION

In this work the vertical and the horizontal forces of interaction in a horizontal magnetic support (HMS) system (also referred to as magnetic levitation or suspension, MLS) are analyzed. The system contains multi-row magnetic strips (permanent magnets) and has a stepped structure as well as a classical structure.

The magnets have a rectangular cross-section and a sufficiently high stability of the magnetization $\vec{M}$ throughout their entire volume ($\vec{M} = \boldsymbol{const}$). The magnetization vector is directed vertically, along both the positive and negative directions of the z-axis.

II. MAGNETIC SUSPENSION (SUPPORT) SCHEMES

Schematic diagrams of the analyzed systems are represented in Figs. 1 to 3 (cross-section of magnetic systems). The length is in the normal direction.

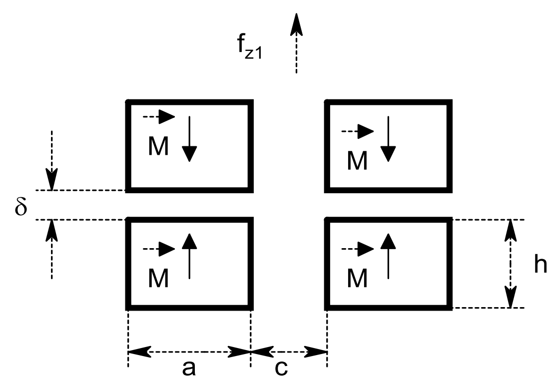

Fig. 1. Classical scheme of MLS.

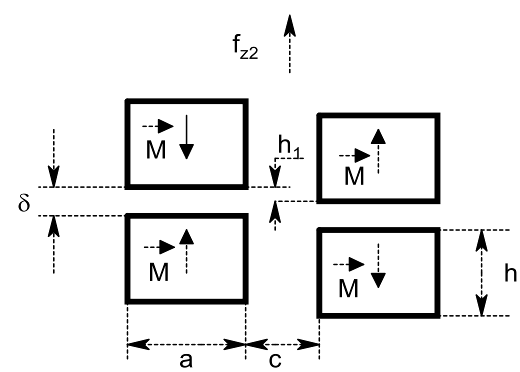

Fig. 2. Scheme with vertical displacement (stepped structure).

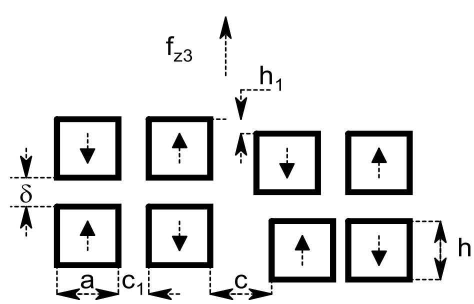

Fig. 3. Combined scheme, classical and stepped structure.

Submitted On: March 15, 2017
Accepted On: July 11, 2017

In all of the depicted schemes, the cross-sectional dimensions of the magnets are much smaller than their lengths (in the normal direction). That is, $a<<l$ and $h<<l$. In the system depicted in Fig. 4, the magnets *1a* and *1b* in the suspended section A are fixed on a the non-ferromagnetic base 2. Magnets *4a* and *4b* of the stationary section B are also fixed on the non-ferromagnetic base 3, like the magnets *1a* and *1b*. The basic working gap is referred to as δ.

The distance between adjacent magnetic strips is equal to *c*. In the figure, arrows are used to indicate the direction of the magnetization $\vec{M}$ of the permanent magnets 1 and 4.

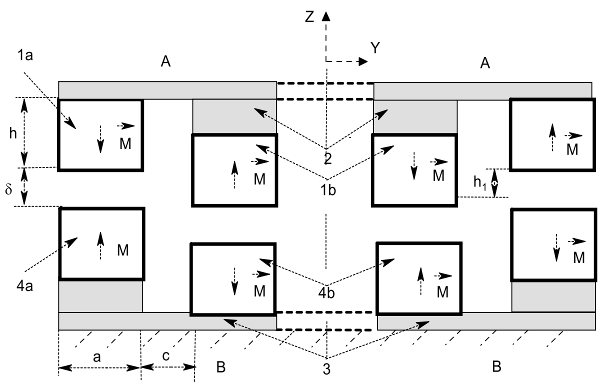

Fig. 4. The symmetrical scheme of magnetic support.

As is evident from the figure, the magnetic strips *1a* and *1b* (or *4a* and *4b*) are moved in the vertical direction to h_1. The expected effectiveness of the suspension system is based on the special features of the cross-interaction of magnets *1b* and *4a* (left and right pairs).

In other words, the effectiveness of the system can be explained by the distribution of the magnetic flux created by these magnets. Thus, the positive effect of the magnetic bearings (suspension) is achieved through the use of leakage flux.

The magnetic system shown in Fig. 3 differs from the system depicted in Fig. 2, in that each of the magnetic strips shown in Fig. 2 has been replaced by two strips with alternating polarities. In the plane YOX, the support system can take the form shown in Fig. 5.

III. INTERACTION ANALYSIS FOR STEPPED SYSTEM

Under certain conditions ($a<<l$), the description of the interaction of the magnetic systems, represented with adequate accuracy in Fig. 5.1, is also valid for the case depicted in Fig. 5.2. Therefore, the interaction analysis of the magnetic systems depicted in Fig. 1 can be carried out only for the linear system (Fig. 5.1).

The forces of vertical and horizontal interaction in the magnetic systems can be determined using the expression for the potential energy of a permanent magnet that is located in an external magnetic field:

$$E_p = \mu_0 \iiint_V \vec{M} \cdot \vec{H} \cdot dV \, , \tag{1}$$

In (1) $\mu_0 = 4\pi \cdot 10^{-7} \left[\frac{H}{m}\right]$, $\vec{M}$ is the magnetization vector (e.g., the magnet *1a* or *1b*) and $\vec{H}(y,z)$ is the vector of magnetic field intensity (of the external magnetic field), created, for example, by the magnet *4a* or *4b*. Integration is performed on the volumes of the magnets which possesses the magnetization $\vec{M}$.

Expressions for the interaction forces of permanent magnets can be obtained using the equation $\vec{F} = -\vec{\nabla} E_p$. For the vertical and horizontal components of the force, this formula gives us:

$$\vec{F}_z = -\hat{z}\frac{\partial E_p}{\partial z}, \qquad \vec{F}_y = -\hat{y}\frac{\partial E_p}{\partial y}\, , \tag{2}$$

where $\hat{y}$ and $\hat{z}$ are the unit vectors of axes y and z respectively.

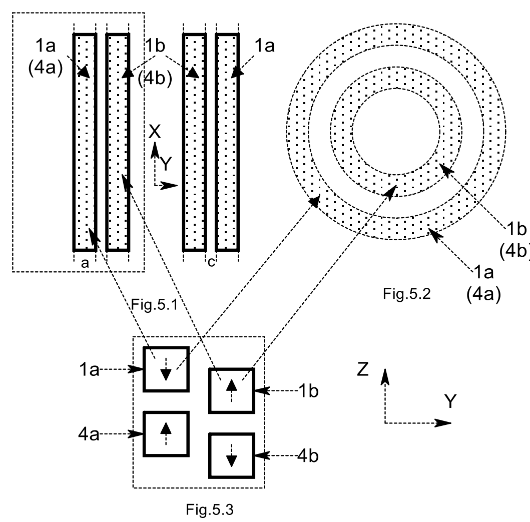

Fig. 5. Details of the stepped magnetic suspension. 5.1: Linear support. 5.2: Locked (circular) support. 5.3: Cross section. *1a* and *1b* – moving magnets of the support system, *4a* and *4b* – stationary magnets.

The efficiency μ_{eff} of the support schemes shown in Figs. 1 to 3, can be estimated using the following expression:

$$\mu_{eff} = f_Z/(mg)\, , \tag{3}$$

in which $f_z \; [N/m]$ is the vertical interaction force for the unit length of the system that includes magnets *1* and *4* and *mg* is the weight per unit length of magnets *1a* and *1b* (or magnets *4a* and *4b*).

To find the interaction force of magnetic systems, we must first determine the magnetic field intensity

$\vec{H}(y,z)$ in those systems. Each magnet in Fig. 4 (e.g., *1a* or *4a*) has a rectangular cross-section and can be represented by two faces (strips), each with a uniformly distributed fictitious magnetic charge with a surface density $\sigma = \pm\mu_0 \cdot M$ [1].

The calculation schemes for the determination of $\vec{H}(y,z)$ referred to Figs. 1 to 3 are shown in Fig. 6.

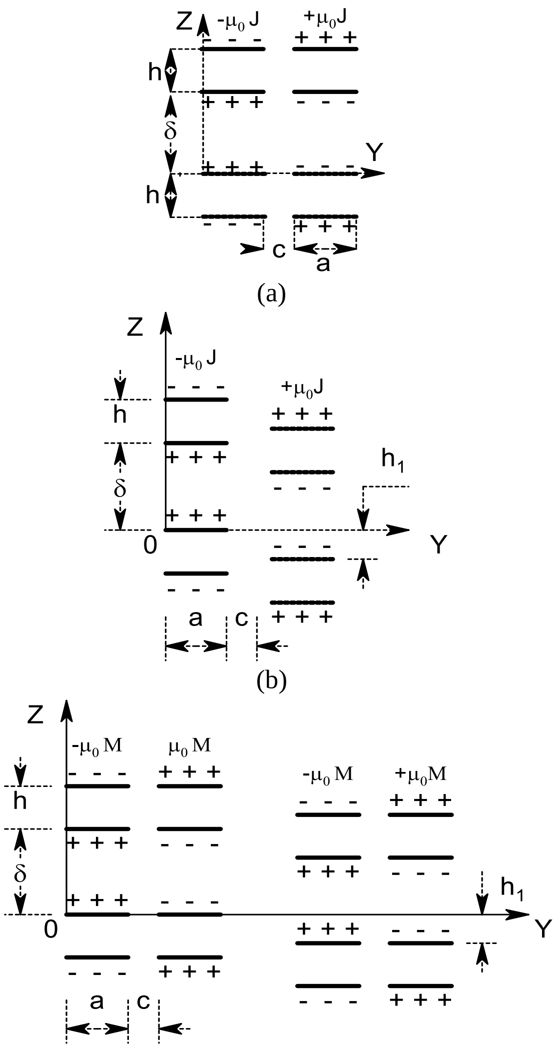

Fig. 6. Calculation schemes corresponding to Figs. 1 to 3.

In accordance with [1, 2] and as shown in Fig. 7, based on the concept of the fictitious magnetic charge, the two-dimensional potential of the magnetic field produced by the "face-charge" at any point, can be expressed as:

$$\phi(y,z) = -\frac{\sigma}{4\pi \cdot \mu_0}\int_0^a \ln\left[z^2 + (y-u)^2\right]\cdot du . \quad (4)$$

The components of the intensity of the magnetic field are determined by the following expressions:

$$H_z(y,z) = -\frac{\partial\phi(y,z)}{\partial z}\ ;\ H_y(y,z) = -\frac{\partial\phi(y,z)}{\partial y} . \quad (5)$$

By substituting (4) into (5), we obtain the expressions for the intensity of the magnetic field at the point $P(y,z)$:

$$H_z(y,z) = \frac{\sigma}{2\pi\mu_0}\left(arctg\,\frac{y}{z} - arctg\,\frac{y-a}{z}\right), \quad (6)$$

$$H_y(y,z) = \frac{\sigma}{2\pi\mu_0}\left[\ln\left(y^2+z^2\right) - \ln\left((a-y)^2 + z^2\right)\right]. \quad (7)$$

The vertical component of the force interaction of two charged surfaces with charge densities $\sigma_1 = \sigma_2 = \sigma$ can be determined using (1) and (2):

$$F_z = \sigma\int_t^{t+a} H_z(y,z)dy . \quad (8)$$

Now, using (6) after a series of transformations, the vertical force per unit length of magnetic systems can be written for the scheme in Fig. 6 (a), as:

$$\begin{aligned} f_{z1} &= 2f_z(0,\delta) - 4f_z(0,\delta+h) + 2f_z(0,\delta+2h) + \\ &-2f_z(a+c,\delta) - 2f_z(a+c,\delta+2h) + \\ &+4f_z(a+c,\delta+h),\quad (N/m), \end{aligned} \quad (9)$$

For the scheme in Fig. 6 (b), we can write,

$$\begin{aligned} f_{z2} &= 2f_z(0,\delta) - 4f_z(0,\delta+h) + 2f_z(0,\delta+2h) + \\ &f_z(a+c,\delta-h_1) + 2f_z(a+c,\delta+h-h_1) + \\ &-f_z(a+c,\delta+2h-h_1) - f_z(a+c,\delta+h_1) + \\ &2f_z(a+c,\delta+h+h_1) - f_z(a+c,\delta+2h+h_1), \end{aligned} \quad (10)$$

where the expression for $f_z(y,z)$ has the following form [2]:

$$\begin{aligned} f_z(y,z) &= \frac{\mu_0 M^2}{2\pi}((y+a)\cdot arctg\,\frac{y+a}{z} + \\ &+(y-a)\cdot arctg\,\frac{y-a}{z} - 2y\cdot arctg\,\frac{y}{a} \\ &+\frac{z}{2}\ln\frac{(z^2+y^2)^2}{(z^2+(y-a)^2)\cdot(z^2+(y+a)^2)}). \end{aligned} \quad (11)$$

Finally the expression of f_{z3} can be similarly derived by properly considering the interactions between the charged surfaces reported in Fig. 6 (c).

Referring to Figs. 1 to 3, the following values of the parameters have been selected for the evaluation of f_{z1} and f_{z2}: $a = 0.02m$, $h = 0.015m$, $h_1 = 0.01 \div 0.02m$ ($\Delta h_1 = 0.002m$), $\delta = 0.005 \div 0.02m$ ($\Delta\delta = 0.0025m$), $c = 0 \div 0.01m$ ($\Delta c = 0.0025m$), $c_1 = 0m$; and $c_1 = 0.005m$.

Some of the results of these calculations are presented in the Table 1 below, in which f_{z1} is the vertical force of interaction (with $h_1 = 0$, ref. to Fig. 1, i.e., the configuration without the step).

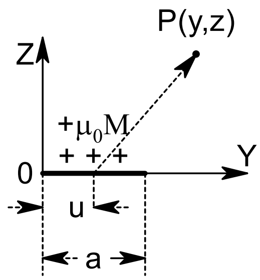

Fig. 7. The calculation scheme for a charged surface of a magnet.

Table 1: Results of the calculations for $\delta = 7.5mm$

$\mu_{eff1} = \frac{f_{z1}}{mg}$	$h_1\ [mm]$	$\frac{f_{z2}}{f_{z1}}$	$\mu_{eff2} = \frac{f_{z2}}{mg}$
10.4	1.0	1.49	15.4
10.4	1.2	1.53	16.0
10.4	1.4	1.54	16.0
10.4	1.6	1.50	15.5
10.4	1.8	1.41	14.6
10.4	2.0	1.24	12.8

A graph of the dependence $f_z(\delta)$ for two cases (with the step (f_{z2}) and without the step (f_{z1})) is presented in Fig. 8.

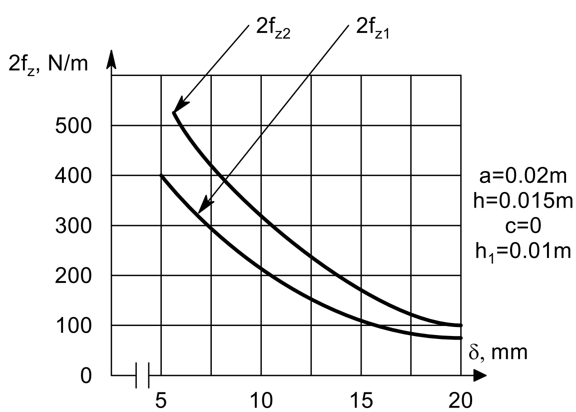

Fig. 8. Graph of the dependence $f_z(\delta)$.

IV. SIDE FORCES ANALYSIS

In the case of two magnetic strips which are offset relative to each other in the horizontal direction by the distance $\tilde{y}$, as shown in Fig. 9, expression (1) becomes:

$$E_p = \mu_0 M \int_0^b dx \int_{\tilde{y}}^{\tilde{y}+a} dy \int_{\delta}^{\delta+h} H_z dz . \tag{12}$$

In the case of the interaction of n magnetic strips, the potential energy of the system is determined by summing the energies Π_{ij} of each i[th] fixed magnet (*4a* or *4b*, Fig. 4, $h_1 = 0$) with each j[th] magnet (*1a*, *1b*) of the moving part of the system, i.e.,

$$E_{p0} = \sum_{i=1}^{n} \sum_{j=1}^{n} \Pi_{ij} . \tag{13}$$

The dependence of side destabilizing forces (acting on a unit length of a magnetic strip) on the value of lateral displacement takes the form:

$$f_y = -\frac{1}{l \cdot n} \cdot \frac{\partial}{\partial \tilde{y}} \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \Pi_{ij}\right), \tag{14}$$

where n is the number of magnetic strips on the mobile (or stationary) portion of the magnetic bearing, and l is the length of the magnetic strip.

Expression (14), which takes into account formulas (6), (7) and (13) for interacting bands (Fig. 5) with two pairs of lateral displacement $\tilde{y}$, allows to record a destabilizing force in the form of:

$$\begin{aligned} f_y = -\mu_0 \frac{M^2}{2\pi} \Big(& \int_{\delta}^{\delta+h} H_z \Big|_{y=\tilde{y}}^{y=\tilde{y}+a} dz + \\ & \frac{1}{2} \int_{\delta}^{\delta+h} H_z \Big|_{y=a+c-\tilde{y}}^{y=-\tilde{y}+2a+c} dz - \frac{1}{2} \int_{\delta}^{\delta+h} H_z \Big|_{y=\tilde{y}}^{y=\tilde{y}+2a+c} dz \Big) \end{aligned} . \tag{15}$$

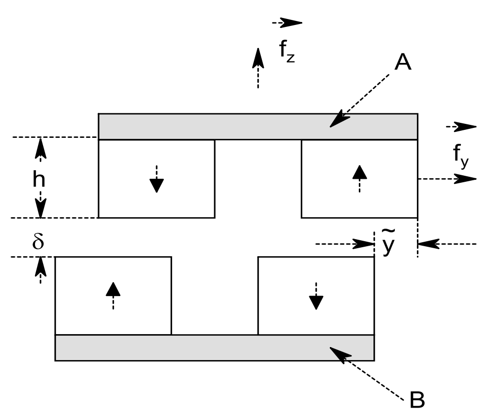

Fig. 9. The classical magnetic support scheme with lateral displacement.

Table 2 shows the results of calculations of the lateral force f_y, dependent on the distance c between the strips, with $l = 1m$, $\delta = 7.5mm$, $a = 60mm$, $h = 50mm$ and $h_1 = 0mm$, that is, the vertical offset (step) is absent. The same table shows the ratio of the vertical and lateral forces, ($\gamma = f_z / f_y$), defined as a measure of system stability (this index can essentially be defined as a destabilizing factor; the greater the value, the less resistant the support system is to tipping or sliding).

Table 3 shows the results of calculating the lateral force, dependent on the side displacement, at $l = 1m$, $\delta = 7.5mm$, $a = 60mm$, $h = 50mm$.

Table 4 shows the calculation of f_y for two and three strips with $l = 1m$, $\delta = 7.5mm$, $a = 60mm$, $h = 50mm$.

Table 2: Dependence of the side force on the distance between the strips: $\tilde{y} = 1mm$, $n = 3$, and $h_1 = 0$

$c\ [mm]$	$f_y\ [N/m]$	$\gamma = f_z / f_y$
5	47.7	19.3
10	52.3	20.0
15	40.7	22.5

Table 3: Dependence of lateral forces on the magnitude of side displacement $\tilde{y}$: $n = 3$, $c = 10mm$

$\tilde{y}\ [mm]$	$f_y\ [N/m]$	$f_y / f_{\tilde{y}=1mm}$
1	44.0	1
2	87.6	2
3	130.7	3
4	173.0	3.9

Table 4: Relationship of side force with number *n* of strips

n	$\tilde{y}\ [mm]$	$f_y\ [N/m]$
2	1	41.0
3	1	44.0
2	2	81.0
3	2	88.0

Figure 10 is a plot of the clearance δ at the lateral displacement $\tilde{y}$, when $a = 64mm$, $h = 56mm$ and $c = 10mm$.

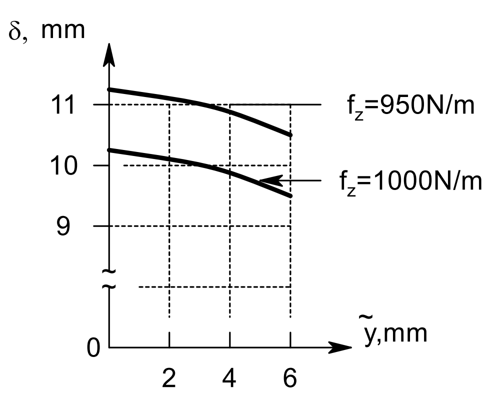

Fig. 10. Loci of the vertical force in the lateral displacement - clearance ($\tilde{y}$ - δ) plane.

V. CONCLUSION

The vertical and horizontal forces of interaction between permanent magnets in a magnetic support system (magnetic suspension, MS) have been investigated. The main results of the investigations are here briefly summarized.

1. The analysis of the obtained data for the stepped MS indicates that the investigated magnetic suspension system (Fig. 2 and Fig. 3) outperforms the classic horizontal system (Fig. 1).

2. The ratio $\gamma = f_z / f_y$ increases with increasing width *a*, indicating a reduction of the instability of the support system in the horizontal plane (Fig. 9).

3. The stability factor γ grows more slowly than the width *a* of the magnetic strip. Doubling $\tilde{y}$ causes $\gamma = f_z / f_y$ to double. The nonlinearity of the relationship $f_y(\tilde{y})$ becomes apparent when $\tilde{y}/a \geq 0.5$. For smaller values of $\tilde{y}$, the lateral force between the magnets can be approximated by $f_y(\tilde{y}) = k\tilde{y}$.

4. Increasing the number of rows of magnets leads to an increase of the lateral force f_y, as was the case for the vertical force f_z.

REFERENCES

[1] W. Baran, "Berechnung von Anziehungs-und Haftkraften fur Magnet emit Feinpolteilung," *Techn. Mitt. Krupp Forsch. - Ber*, vol. 21, pp. 72-83, 1963.

[2] A. E. M. Frishman and M. I. Umanov, "Analysis of vertical and horizontal interaction forces in permanent magnets maglev systems," *Vestnik Dnepropetrovskogo nazionalnogo Universiteta Zeleznodoroznogo Transporta*, vol. 2, no. 41, pp. 136-141, 2012.