Algorithm for Accounting for Inner Damping in a Computer Model of Dynamics of a Flexible Rotor on Active Magnetic Bearings

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Abstract **─** An algorithm for accounting for inner damping in an actively developed computer model of dynamics of a flexible rotor on active magnetic bearings is presented. The algorithm is illustrated by applying it for analyzing the effects of amplitude-independent inner damping on rotor dynamics when residual imbalance is present.

Index Terms **─** Active magnetic bearing, dynamics influenced by inner damping, flexible rotor, inner damping.

I. INTRODUCTION

One of popular applications of suspension systems based on active magnetic bearings (AMB) is supporting big size and weight flexible rotors of complex design (as in [1, 2]). Such multi-ton flexible heterogeneous rotors meet the criteria formulated in [3] that define complex unique systems. Such rotors comprise dissimilar machines: turbines, power generators, compressors, etc. Each AMB has its own control system interacting with the systems for other bearings via the rotor itself. Numerous forces influencing rotor behaviour [4] – such as weight, imbalance, gyroscopic, thermomechanical, gas-dynamic and electromagnetic effects – excite several simultaneous mode shapes in a flexible rotor, a combination of which can result in substantially different movement patterns for different rotor parts.

For the studies required both during design and operation phases of such systems, a computer model of dynamics of a flexible heterogeneous rotor on AMB is being actively developed. One of interests of development is extending the list of available forces that influence the system by adding new, yet unaccounted, forces that have considerable effect on the rotor dynamics.

Paper [5] presents formulas for generalized forces which influence dynamics of a flexible rotor on AMB: imbalance force, gyroscopic force, gravity, magnetic and circulation forces caused by electromagnetic interaction in the generator, exciter and motor as well as gas-dynamic forces in turbines, compressors and labyrinth seals of their shafts.

Inner damping can have substantial effects on rotor dynamics [6, 7], especially in transitional modes of starting and stopping the rotation when frequency passes critical values. This paper aims at designing an algorithm for accounting for inner damping in an actively developed computer model of dynamics of a flexible rotor on AMB. The introduced algorithm is illustrated by computations demonstrating the effect of inner damping on dynamics of a flexible rotor on AMB when residual imbalance is present.

II. DISCRETE MATHEMATICAL MODEL OF ROTOR DYNAMICS

The actively developed computer model of dynamics of a flexible rotor on AMB is a discrete mathematical model of rotor expressed in a computer code [5]. The model includes a mechanical model, a control system model and models of forces affecting the behaviour of the rotor on AMB.

The implementation of the aforementioned model is oriented on performing the calculations in a parallelized way, either using conventional CPU parallelization or utilizing the power of GPGPU which is now becoming more and more used in such modellings [8, 9].

Mechanical model is based on the equations and results of studying dynamics of flexible rotors [7]. A otor is considered to be a resilient heterogeneous rod. Specific characteristics of the dynamics of the rotor on AMB are determined by flexural vibrations, which are the objective of this paper.

The model of the rotor is a series of homogeneous cylinder parts combined into one mechanical model by two types of connections: rigid coupling and flexible clutches. Flexural vibrations are described using Timoshenko beam model [10]. The solution of the initial distributed problem of determining flexural vibrations is determined by four functions describing linear and angular motions of the rotor as well as inner stresses and moments. Transition to a discrete model is based on basis functions expansion of the solution. The choice of basis functions depends on the problems being solved. The presented model proposes to generate the required laws

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of variation of forces in the AMB control system, including those that are linearly dependent on rotor displacement [11, 12]. As a result, basis functions for the problem are chosen to be orthogonal eigenmodes of the oscillations of a rotor on resilient bearings with the assigned rigidity at the cross-sections where the AMBs were situated.

To obtain resulting equations of the mechanical model of a flexible rotor [13], Lagrange equations are used. Equations describing rotor dynamics in a global fixed coordinate system in the two orthogonal crosswise directions are composed for vectors of generalized coordinates C_i. The generalized coordinates are eigenmode expansion coefficients. The dimension of C_i – n – is the number of generalized coordinates, i.e., the number of eigenmodes accounted for when approximating the solution. The dynamics equations in matrix form for C_i are as follows:

$$m\frac{d^2C_i}{dt^2}+m\ \Omega\ C_i=F_i,\ i=\overline{1,2}, \tag{1}$$

where m is rotor mass; Ω is a diagonal $n\times n$ matrix, its elements being squares of eigenfrequencies of the rotor; F_i are vectors of generalized forces (with the dimension of n) acting in two orthogonal crosswise directions. Each F_i force is the resultant force of all the forces accounted for in the model (1) of different natures.

The control system model [5] accounts for the discrete nature of polling displacement sensors, operation of current amplifiers with dead space and relay parts, a lag in forming of control signal, the nonlinear dependence of the AMB forces on the current in the coils and a gap between the magnets and the rotor surface.

III. MODEL OF FORCES CAUSED BY INNER DAMPING

Inner damping includes both energy dissipation in the material and structural damping in the joints. It destabilizes rotor dynamics, so taking inner damping into account is essential when constructing a model of dynamics.

Numerous studies [6, 14-16] show that in a wide range of frequencies the amount of energy dissipation does not depend on the strain rate, but rather on type of the stressed state, strain level and on the temperature of material. Structural damping in the joints has similar properties. Currently there exist numerous theories of inner damping that in certain conditions are able to account for the main features of the energy dissipation process. Those theories use relations between strain and stress tensors in the process of oscillations (including nonstationary ones). In general, those relations are nonlinear and ambiguous (e.g., of a hysteresis type). Most theories and models of inner energy dissipation that are conformant to experiments are either too complex for practical application or aimed at solving the problem with special conditions (stable oscillations, single-frequency oscillation, linear approximations) [14, 16].

The presented approach for accounting for inner damping is based on two assumptions: a) oscillation behaviour mainly depends on the amount of energy dissipated during one oscillation cycle (the area of hysteresis loop), and detailed characteristic of stress caused by deformation (the hysteresis loop shape) do not influence behaviour much; b) forces caused by inner damping are of small value, so they cannot lead to any noticeable interaction of different oscillation processes in model (1).

In the introduced model it is assumed that generalized forces caused by inner damping are independent for all generalized coordinate components ($\{q_k\}$, k=1,…,n) of vector of generalized coordinates Q that describes crosswise displacement of the rotor in the coordinate system fixed to the rotor (and rotating with the rotor). For each generalized coordinate a phenomenological model of friction damping is used (model by Korchinsky, model by Leonov and Bezpalko) [15]. Keeping this in mind, generalized force caused by inner damping can be written as follows:

$$r_k=-s_k\cdot\chi\cdot\left|q_k\right|^{\alpha}sign\frac{dq_k}{dt},\ k=\overline{1,n}, \tag{2}$$

where r_k is a component of generalized damping force R; s_k is effective mechanic rigidity (as in mechanical model (1)) along the generalized axis; χ, α are parameters describing the relation of energy dissipation and rotor oscillation level.

If deformations along generalized axis q_k are cyclic and have amplitude u_k, then energy losses ΔW during one cycle are described as follows:

$$\Delta W=4s_k\cdot\chi u_k^{\alpha+1}/\left(\alpha+1\right),\ k=\overline{1,n}. \tag{3}$$

Total oscillation energy is $W=s_k\cdot u_k^2/2$, and, taking (3) into account, relative energy dissipation ψ will be:

$$\psi=\Delta W/W=8\cdot\chi u_k^{\alpha-1}/\left(\alpha+1\right),\ k=\overline{1,n}. \tag{4}$$

Considering that logarithmic decrement δ is coupled with the relative energy dissipation: $\psi=2\delta$, Equation (4) makes it possible to determine parameters χ and α experimentally from the measured logarithmic decrements.

For α= 1, it follows from (4) that χ = δ/2, thus (2) can be transformed to:

$$r_k=-s_k\cdot\delta\cdot\left|q_k\right|sign\frac{dq_k}{dt}/2,\quad k=\overline{1,n}. \tag{5}$$

Equation (5) describes components of generalized forces caused by inner damping when damping is assumed amplitude-independent.

The dependence of inner damping on the oscillation amplitude of a flexible rotor can be taken into account by identifying the parameters of inner damping based on phenomenological model of friction damping.

To describe deformations of a flexible rotor, its motion in the coordinate system fixed to the rotor is examined. For the vectors of the generalized coordinates Q_i (i=1,2) characterizing rotor motion in the moving coordinates, the following equations hold:

$$\begin{aligned} Q_1 &= C_1 \cdot \cos\varphi + C_2 \cdot \sin\varphi; \\ Q_2 &= -C_1 \cdot \sin\varphi + C_2 \cdot \cos\varphi, \end{aligned} \tag{6}$$

where φ is rotor angle – the angle between axes of fixed and moving coordinate systems.

Forces R_i caused by inner damping in the moving coordinates are expressed as follows:

$$R_i = -\frac{m}{2} D \cdot \Omega \cdot H(Q_i) \quad (i = 1,2), \tag{7}$$

where D is a diagonal matrix of logarithmic decrements for eigenmodes; $H(X)$ is a vector function with dimension n, with its components h_k defined as:

$$h(x_k) = |x_k| sign \frac{dx_k}{dt},\ k = \overline{1,n}\ ,$$

where x_k are the components of n-dimensional vector X, argument of $H(X)$ function.

When expressed in a fixed coordinate system, generalized forces of inner damping N_i produce additions to the right-hand member of original Equations (1):

$$\begin{aligned} N_1 &= R_1 \cos\varphi - R_2 \sin\varphi, \\ N_2 &= R_1 \sin\varphi + R_2 \cos\varphi. \end{aligned} \tag{8}$$

Thus, accounting for inner energy dissipation and structural damping during rotor deformation could be reduced to adding generalized forces (8) to mathematical model of rotor dynamics (1). Implementation of the algorithm (1) could be summarized as following - at each integration step we sequentially execute 3 procedures:

1) Using known vectors of generalized coordinates and speeds in the fixed coordinate system, rotor angle and Equation (6) compute respective vectors in the coordinate system attached to the rotor.
2) Using forces Equation (7) compute vectors of generalized forces caused by inner damping in the coordinate system attached to the rotor.
3) Basing on (8) compute respective generalized forces' vectors in the fixed coordinate system.

IV. A CASE STUDY OF EFFECTS OF INNER DAMPING ON DYNAMICS OF A FLEXIBLE ROTOR

As a case study proposed algorithm was applied to a flexible vertical rotor of the generator part of the RSM (Rotor Scale Model) test bench [17] consisting of two radial AMBs with the length of 5.4 m and the mass of 640 kg. Default control law is a "linear" one that specifies AMB force to be linearly proportional to the displacements and speeds of the rotor [11]. The lowest eigenfrequencies of oscillations of rotor on AMB are 7.5 Hz, 8.2 Hz, 15.8 Hz, 39.3 Hz and 77.5 Hz.

The first two frequencies mostly correspond to rotor oscillations as a rigid body; the others correspond mostly to flexural oscillations.

The effect of inner damping on rotor dynamics was studied for the case of residual imbalance. Inner damping is characterized with logarithmic decrement δ. Figure 1 depicts rotor displacements in the section of the upper AMB as a function of the rotation frequency in the acceleration mode with a constant angular acceleration of 0.1 Hz/sec for δ=0.05 and δ=0.10. Here and below the black curves correspond to logarithmic decrement δ=0.05 and the gray ones correspond to δ=0.10.

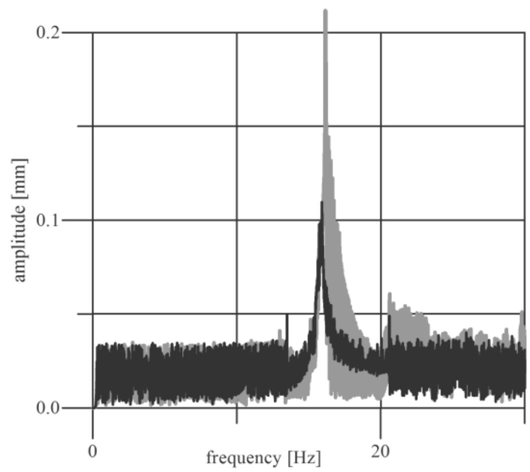

Fig. 1. Rotor displacement as a function of rotation frequency.

Figures 2-4 present rotor oscillations for the rotation frequencies of 10 Hz, 22 Hz and 25 Hz. For subcritical frequencies (under 15 Hz), rotor displacement for a given imbalance is almost constant and is determined by dead space in the current amplifiers of the AMB control system. In the presence of resonance and in the supercritical frequency region the displacement grows when internal damping increases. The main factor causing the growth is the increase of the amplitude of the first flexural eigenmode with the rotation frequency.

Figure 5 depicts oscillation spectrum for the rotation frequency of 23 Hz. The value of the peak corresponding to this frequency in the oscillation spectrum as a function of rotation frequency is shown in Fig. 6.

Rotor oscillations at the frequency of first flexural eigenmode are constantly present because of random perturbations generated in AMB control system which includes relay parts. When inner damping increases, a sharp increase of the oscillation amplitude is observed at supercritical rotation frequencies, which effectively describes the destabilization effect of inner damping. Such effects are in good agreement with known theoretical and experimental results for rotors on conventional slider bearings: the higher inner damping is, the higher is the hazard of exciting flexural oscillations of the rotor [6, 7].

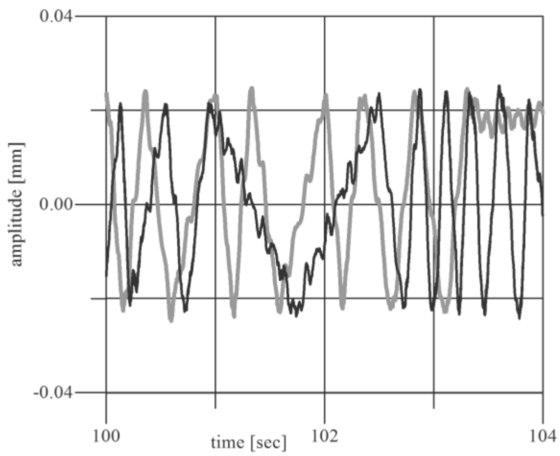

Fig. 2. Rotor oscillations for the rotation frequency of 10 Hz.

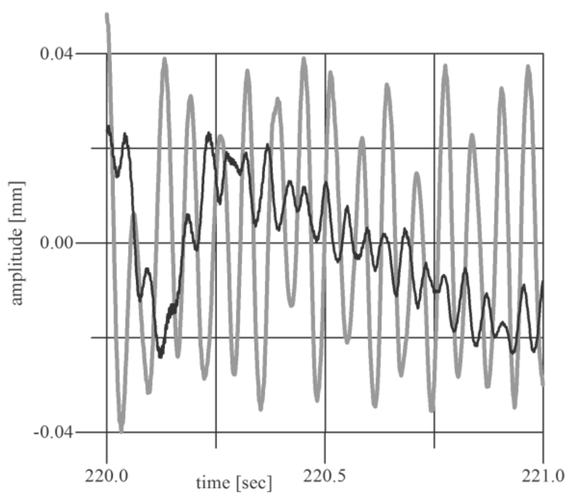

Fig. 3. Rotor oscillations for the rotation frequency of 22 Hz.

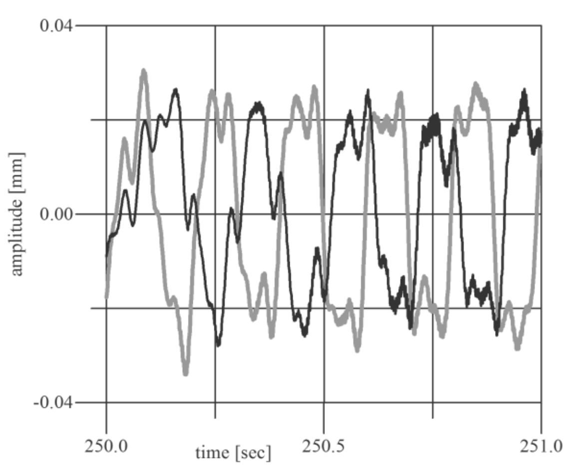

Fig. 4. Rotor oscillations for the rotation frequency of 25 Hz.

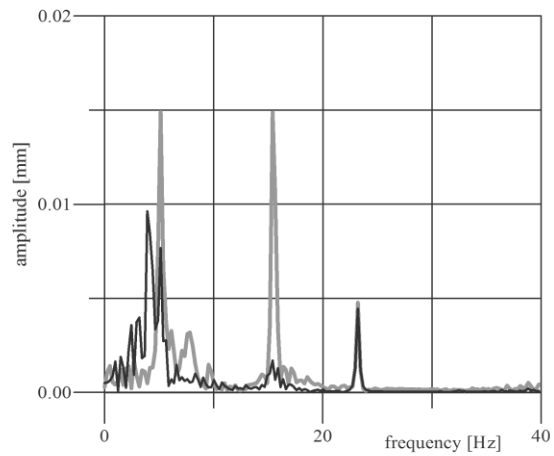

Fig. 5. Rotor oscillation spectrum at the rotation frequency of 23 Hz.

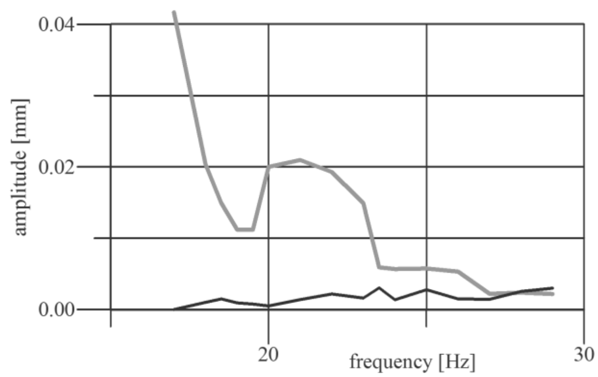

Fig. 6. The peak in the oscillation spectrum caused by the frequency of first flexural eigenmode as a function of rotation frequency.

V. CONCLUSION

An algorithm for accounting for inner damping in an actively developed computer model of dynamics of a flexible rotor on AMB is presented. The model is a computer program of a discrete mathematical model of the rotor. The application of the algorithm is illustrated by analyzing the effect of amplitude-independent inner damping on dynamics of a flexible rotor in presence of residual imbalance. The results of modelling agree with known data for rotors on conventional slider bearings the higher inner damping is, the higher is the hazard of excitation of flexural rotor oscillations at supercritical rotation frequencies. The software implementation of the algorithm allows for sufficiently accurate account for inner damping effect in the computer model of dynamics of a flexible rotor on AMB. It also almost does not increase computation time which is very important for conducting multi-variant numerical experiments which are essential for design and operation of such complex rotor-based systems.

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